Pregroup Grammar. Theory and Applications

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1. Introduction

The Syntactic Calculus (Lambek 1958, 1961), developed from earlier systems by Ajdukiewicz and Bar-Hillel, allows compound types to be built up, starting from a finite set of basic types, by three operations:

$$A{\otimes}B\ ,\ A/B \text{ and } A\backslash B\ ,$$

subject to the following axioms and rules of inference:

$$(AB)C \leftrightarrow A(BC)\ ,$$

$$AB \rightarrow C \ \text{ iff } \ A \rightarrow C/B \ \text{ iff } \ B \rightarrow A\backslash C\ ,$$

where $\rightarrow$ is a deduction symbol satisfying reflexivity and transitivity.

From now on we abbreviate $A{\otimes}B$ as AB.

Later Lambek introduced a constant 1 satisfying

$$A1 \leftrightarrow A \leftrightarrow 1A \ .$$

Note that the Syntactic Calculus with 1 is not a conservative extension of that without 1.

For example, $A/(B/B) \rightarrow A$ is provable in the former but not in the latter.

Lambek, J. (1995b). Bilinear logic in algebra and linguistics. In J. Y. Girard et al. (eds.)(1995), *Advances in Linear Logic*, 43–60, Cambridge University Press, Cambridge.

Lambek, J. (1999a). Deductive systems and categories in linguistics. In H.J. Ohlback and U. Reyle (eds.)(1999), *Logic, Language and Reasoning, Essays in honour of Dov Gabbay*, Kluwer, Dordrecht, 279-294.

In the Syntactic Calculus you prove a number of theorems such as:

$$(A/B)\ B \rightarrow A \qquad \textit{Ajdukiewicz's law}\ ,$$

$$B \rightarrow (A/B)\backslash A \qquad \textit{type raising}\ ,$$

$$(A/B)(B/C) \rightarrow A/C \qquad \textit{composition}\ ,$$

$$A/B \rightarrow (A/C)/(B/C) \qquad \textit{Geach's law}\ ,$$

$$(C/B)/A \leftrightarrow C/(AB) \qquad \textit{Curry's law}\ ,$$

as well as their mirror images such as:

$$B\ (B\backslash A) \rightarrow A \quad \text{etc.}$$

We have also

$$(A\backslash B)/C \leftrightarrow A\backslash(B/C)\ ,$$

hence we often write $A\backslash B/C$ for either side,

and the following rules of inference indicating what categorists call *functoriality*:

$$\frac{A \rightarrow B \quad C \rightarrow D}{AC \rightarrow BD} \qquad \frac{A \rightarrow B \quad C \rightarrow D}{A/D \rightarrow B/C}$$

and the mirror dual of the latter.

Under the influence of Girard (1987), Abrusci (1991) and Lambek (1993) developed a non-commutative version of his linear logic, called

> non-commutative linear logic (Abrusci)
>
> or classical bilinear logic (Lambek).

It can be obtained from the Syntactic Calculus by adjoining a constant symbol 0 satisfying:

$$0/(A\backslash 0) \leftrightarrow A \leftrightarrow (0/A)\backslash 0$$

where we can also write

$$A\backslash 0 = A^r, \quad 0/A = A^\ell \ .$$

One can show that

$$(B^r A^r)^\ell \leftrightarrow (B^\ell A^\ell)^r \ ,$$

for which it is convenient to write $A \oplus B$, or simply A+B. We may think of $\oplus$ as the De Morgan dual of $\otimes$: it corresponds to what Girard calls "par" [1]. Here are some theorems of classical bilinear logic, which had been anticipated by Grishin (1983):

$$\begin{array}{rcl} & 1^r \leftrightarrow 0 \leftrightarrow 1^\ell & \\ & A+0 \leftrightarrow A \leftrightarrow 0+A & \\ & (A+B)+C \leftrightarrow A+(B+C) & \\ A^\ell A \rightarrow 0 & , & AA^r \rightarrow 0 \\ 1 \rightarrow A+A^\ell & , & 1 \rightarrow A^r+A \\ A/B \leftrightarrow A+B^\ell & , & B\backslash A \leftrightarrow B^r+A \\ (A+B)\,C \rightarrow A+BC & , & C\,(B+A) \rightarrow CB+A \end{array}$$

The last two are the *mixed associative* laws of Grishin

[1] The duality relation between $\otimes$ and $\oplus$ plays a crucial role in non-commutative linear logic (or bilinear logic), as shown in Abrusci (1991).

2. Compact Bilinear Logic

Lambek observed that there was a simplification if one assumed that

$$\mathrm{A{+}B} \leftrightarrow \mathrm{AB} \quad , \quad 0 \leftrightarrow 1 \; .$$

The word "compact" had been used e.g. by Barr (1979) to describe this situation in a categorical context, though then still restricted to the commutative case. Finally, it was realized that *compact* bilinear logic allowed a simpler description as follows:

$$(\mathrm{AB})\mathrm{C} \leftrightarrow \mathrm{A}(\mathrm{BC}) \quad , \quad \mathrm{A}1 \leftrightarrow \mathrm{A} \leftrightarrow 1\mathrm{A}$$
$$\mathrm{A}\,\mathrm{A}^r \to 1 \to \mathrm{A}^r\mathrm{A} \quad , \quad \mathrm{A}^\ell\,\mathrm{A} \to 1 \to \mathrm{A}\,\mathrm{A}^\ell \; .$$

Barr, M. (1979). ∗-*Autonomous Categories*. Springer LNM 752, Berlin.

Models in which $\rightarrow$ stands for a partial order are called "pregroups";

they reduce to groups if the order is discrete, that is, is the equality relation, and to partially ordered groups in the so-called *cyclic* case when $A^{\ell} \leftrightarrow A^{r}$.

If however the arrows are allowed to stand for morphisms in a category, one would also demand that the above occurrences of $\leftrightarrow$ represent isomorphisms and that the composite arrows

$$A \rightarrow A\,A^{\ell}A \rightarrow A \quad , \quad A \rightarrow A\,A^{r}A \rightarrow A$$

are identity arrows, making A^{ℓ} the left adjoint and A^{r} the right adjoint of A in a 2-category.

3. Definition of Pregroup

A *pregroup* $\{G, \,.\, , 1, {}^{\ell}, {}^{r}, \rightarrow\}$ is a partially ordered monoid in which each element a has a

$$\textit{left adjoint } a^{\ell} \quad \text{and} \quad \text{a} \quad \textit{right adjoint } a^{r}$$

such that

$$a^{\ell}\, a \rightarrow 1 \rightarrow a\, a^{\ell}$$

$$a\, a^{r} \rightarrow 1 \rightarrow a^{r}\, a$$

the dot "." stands for multiplication with unit 1, and the arrow denotes the partial order.

In linguistic applications the symbol 1 stands for the empty string of types and multiplication is interpreted as concatenation.

Adjoints are unique and we prove

$$1^\ell = 1 = 1^r \,,$$

$$(a \cdot b)^\ell \;=\; b^\ell \cdot a^\ell \quad , \quad (a \cdot b)^r \;=\; b^r \cdot a^r \;,$$

$$\frac{a \longrightarrow b}{b^\ell \longrightarrow a^\ell} \quad , \quad \frac{a \longrightarrow b}{b^r \longrightarrow a^r} \quad , \quad \frac{b^\ell \longrightarrow a^\ell}{a^{\ell\ell} \longrightarrow b^{\ell\ell}} \quad , \quad \frac{b^r \longrightarrow a^r}{a^{rr} \longrightarrow b^{rr}} \quad .$$

The following also hold

$$a^{r\ell} = a = a^{\ell r} \;,$$

$$a^{\ell\ell}\; a^\ell \longrightarrow 1 \longrightarrow a^\ell\; a^{\ell\ell} \;,$$

$$a^r\; a^{rr} \longrightarrow 1 \longrightarrow a^{rr}\; a^r \;,$$

A pregroup is *freely generated* by a partially ordered set of *basic* types. From each basic type a we form *simple* types by taking single or repeated adjoints:

$$\ldots\ a^{\ell\ell},\ a^{\ell},\ a,\ a^{r},\ a^{rr}\ldots$$

Compound types or just *types* are strings of simple types. We assign to each word or word form in the dictionary of the language under investigation one (or more) types. The only computations required are *contractions* (C) and *expansions* (E) :

$$\begin{array}{ll} (\mathsf{C}) & a^{\ell}\ a \rightarrow 1\ ,\quad a\ a^{r} \rightarrow 1\ , \\ (\mathsf{E}) & 1 \rightarrow a\ a^{\ell}\ ,\quad 1 \rightarrow a^{r}\ a\ , \end{array}$$

where a is a simple type. For the purpose of sentence verification expansions are not needed, but only contractions, combined with some rewriting induced by the partial order.

Example	
basic types	$\mathrm{s}\ ,\ i\ ,\ \pi\ ,\ \mathrm{o}\ ,\ \omega,\ \lambda$
compound types	$(\pi_1^r\ \mathrm{s}_1\ \mathrm{o}^{\ell})\ ,\ (i\ \lambda^{\ell})\ ,\ i\ \omega^{\ell}\mathrm{o}^{\ell}$

4. Pregroup grammar

Developing a pregroup grammar for a natural language consists in two main steps:

(i) assign one or more (basic or compound) types to each word in the dictionary;

(ii) check the grammaticality and sentencehood of a string of words by a calculation on the corresponding types

where the only rules involved are :

contractions,

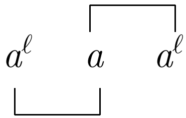
ordering postulates taking the form $\alpha \to \beta$ (α, β basic types)

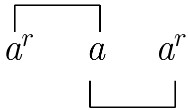
and appropriate conditions introduced in the lexicon, called **metarules**.

5. Contractions and Expansions

$$(\text{Contractions}) \qquad a^{\ell}\ a \longrightarrow 1\ , \quad a\ a^{r} \longrightarrow 1\ ,$$

$$(\text{Expansions}) \qquad 1 \longrightarrow a\ a^{\ell}\ , \quad 1 \longrightarrow a^{r}\ a\ ,$$





6. Ordering postulates

The partial order can be restricted by (language specific) *ordering postulates* that can be positive

$$a \longrightarrow \overline{a} \longrightarrow \overline{\overline{a}}$$

or negative

$$b \not\longrightarrow \overline{b} \quad , \quad \tilde{b} \not\longrightarrow c$$

The bar notation is inspired to the standard X-bar theory, the types with bars are defined as projections or super types of the basic types.

For example, the postulates

$$\hat{\pi}_3 \longrightarrow \pi_3 \text{ but } \pi_3 \not\longrightarrow \hat{\pi}_3 \quad , \quad \hat{o} \longrightarrow \text{o but o} \not\longrightarrow \hat{o}$$

control the calculation of unbounded dependencies in interrogative clauses.

7. Unbounded dependencies

Interrogative pronouns (in a number of European languages such as English, French, Italian) are assigned the types:

$\mathrm{q}\ \hat{\pi}_3^{\ell\ell}\ \mathrm{q}^{\ell}$ *who* subject, nominative ;

$\mathrm{q}\ \hat{o}^{\ell\ell}\ \mathrm{q}^{\ell}$ *whom* direct object, accusative .

The basic types $\hat{\pi}_3^{\ell\ell}$, $\hat{o}_3^{\ell\ell}$ occurring within the types assigned to the interrogative pronouns are related to a missing argument in the context following the pronoun (an unbounded dependency or a *trace* in the sense of Chomsky)

8. Direct questions and Wh-questions

he likes her

$$\pi_3 \; (\pi_3{}^r \; \mathrm{s}_1 \; \mathrm{o}^\ell) \; \mathrm{o} \;\; \longrightarrow \mathrm{s}_1$$

does he like her?

$$(\mathrm{q}_1 \;\; \imath^\ell \;\; \pi_3^\ell) \; \pi_3 \; (\imath \; \mathrm{o}^\ell) \; \mathrm{o} \;\; \longrightarrow \mathrm{q}_1$$

whom does he like – ?

$$(\mathrm{q} \;\; \hat{o}^{\ell\ell} \;\; \mathrm{q}^\ell) \; (\mathrm{q}_1 \;\; \imath^\ell \;\; \pi_3^\ell) \; \pi_3 \; (\imath \; \mathrm{o}^\ell) \;\; \longrightarrow \mathrm{q}_1$$

9. Features

Numerical indices are assigned to simple types to express featural information such as: person, gender, number, case for nouns, noun phrases and adjectives, tense, person for verbs.

10. Templates

$$C^{a}_{tp}(V) = \left[\begin{array}{l|l|l} a = ASPECT & t = TENSE & p = PERSON \\ \hline Basic : 1 & pres.ind. : 1 & first : 1 \\ Perfect : 2 & pres.subj. : 2 & second : 2 \\ Passive : 3 & imperf.ind. : 3 & third : 3 \\ & imperf.subj. : 4 & fourth : 4 \\ & future : 5 & fifth : 5 \\ & & sixth : 6 \end{array}\right]$$

$$\mathrm{D}^{\mathrm{g}}_{\mathrm{nc}}(\mathrm{A}) = \left[\begin{array}{l|l|l} g = GENDER & n = NUMBER & c = CASE \\ \hline Masculine : 1 & singular : 1 & nominative : 1 \\ Feminine : 2 & plural : 2 & genitive : 2 \\ Neuter : 3 & & dative : 3 \\ & & accusative : 4 \\ & & ablative : 5 \end{array}\right]$$

$$\mathrm{D}^{\mathrm{g}}_{\mathrm{nc}}(\mathrm{N}) = \left[\begin{array}{l|l|l} g = GENDER & n = NUMBER & c = CASE \\ \hline Masculine : 1 & singular : 1 & nominative : 1 \\ Feminine : 2 & plural : 2 & genitive : 2 \\ Neuter : 3 & & dative : 3 \\ & & accusative : 4 \\ & & ablative : 5 \end{array}\right]$$

11. Metarules

METARULE 1. If $\mathrm{C}^1_{tp}(\mathrm{V})$ has type $\ldots\ \mathrm{o}^r_4\ \pi^r_p\ \mathrm{s}_t$, then $\mathrm{C}^2_{tp}(\mathrm{V})$ has the same type, but $\mathrm{C}^3_{tp}(\mathrm{V})$ has type $\ldots\ \mathrm{o}^r_5\ \pi^r_p\ \mathrm{s}_t$, provided o^r_5 occurs only once.

METARULE 2. If the verb form $\mathrm{C}^a_{tp}(\mathrm{V})$ has been assigned type $\mathrm{x}^r_1 \ldots \mathrm{x}^r_n\ \mathrm{s}_t$, then

(1) any x^r_i can be omitted ,
(2) x^r_i and x^r_j can be permuted ,
(3) any x^r_i on the left of s_t can be moved to the right as x^ℓ_i .

METARULE 3. An adjective of type a_{gnc}, also has types $(n^r_{gnc}\ n_{gnc})$ and $(n_{gnc}\ n^\ell_{gnc})$.

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