Growth rates constrained by internal and external imbalances and the role of relative prices: Empirical evidence from Portugal.

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Abstract

Thirlwall's Law (Thirlwall 1979) considers that growth can be constrained by the balance-of-payments when the current account is in permanent deficit. The Law focuses on external imbalances as impediments to growth and does not consider the case where internal imbalances (budget deficits or public debt) can also constrain growth. The recent European public debt crisis shows that when internal imbalances are out of control they can constrain growth and domestic demand in a severe way. Recently, Soukiazis E., Cerqueira P., and Antunes M. (2012) developed a model – hereafter the SCA model - that takes into account both internal and external imbalances but where relative prices do not play any role on the pace of economic growth. The aim of this paper is to extend the SCA model by relaxing this assumption and introducing explicitly relative prices in it. The model is tested for Portugal which recently fell into a public debt crisis with serious negative consequences on growth. It is shown that our new model makes a significant improvement in predicting actual growth in Portugal. Our empirical analysis reveals that policies aiming at reducing internal and external imbalances and financing public debt with lower cost will help the country to grow faster.

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1. Introduction

Thirlwall (1979) developed a simple model that determines the long run rate of growth of an economy consistent with the balance-of-payments equilibrium. According to this rule, actual growth can be predicted by the ratio of export growth to the income elasticity of demand for imports[1]. There are two main controversial assumptions on the model: balance-of-payments equilibrium (on current account) and relative prices or real exchange rates remain constant in the long term analysis. According to what became known as Thirlwall's Law, no country can grow faster than its balance of payments equilibrium growth rate, unless it can continuously finance external deficits by capital inflows. Growth is constrained by external demand, and balance-of-payments disequilibrium on the current account can be a serious obstacle to higher growth when it cannot be financed by available foreign resources. Another crucial implication of the model is that it is income and not relative prices that adjust to bring the economy back to equilibrium.

Later on, Thirlwall and Hussain (1982) revised the model relaxing the assumption that the balance-of-payments is initially in equilibrium. Since countries can run current account deficits, capital inflows can be included in the model to determine the long term growth rate. This model has shown to be more realistic especially for developing countries where external imbalances can be sustained by capital inflows that alleviate the pressure on external payments. A large number of empirical studies emerged testing the validity of Thirlwall's Law or criticising the basic assumptions that it relies on. Among others, Moreno-Brid (1998-99), McCombie and Thirlwall (1994) and recently Blecker (2009) have made valuable contributions discussing and criticising the underlying implications of the Law.

The hypothesis of constant relative prices has been criticized widely in empirical literature (e.g. McGregor and Swales, 1985; 1991; Alonso and Garcimartín, 1998-99; López and Cruz, 2000). But in most studies in this field, relative prices have been shown to be statistically insignificant and even when they are significant the price elasticities with respect to imports and exports are very low in magnitude when

[1] Thirwall's Law is given by $\dot{y} = \frac{\dot{x}}{\pi}$ where $\dot{y}$ is the growth of domestic income, $\dot{x}$ is the growth of real exports, and π is the income elasticity of the demand for imports. To obtain this simple form relative prices are assumed to be constant and balance of payments is in equilibrium (on the current account).

compared to the income elasticities, showing that imports and exports are less sensitive to price changes than to income changes. Alonso and Garcimartín (1998-99) showed that the assumption that prices do not matter in determining the equilibrium income is neither a necessary nor a sufficient condition to assert that growth is constrained by the balance-of-payments. The empirical evidence seems to support the idea that income is the variable that adjusts to equilibrate external imbalances, implying therefore that growth is indeed balance-of-payments constrained. Blecker (2009) also stressed that it is safe to conclude that the longer the time period considered, the more likely it is that relative prices remain constant. On the other hand, increasing capital inflows can at most be a temporary way of relaxing the balance-of-payments constraint, since they do not allow a country to grow at the export-led cumulative growth rate in the long term. What matters in the long-term analysis of growth is the growth of exports.

Although Thirlwall's model has been modified to include capital flows and foreign debt, these studies have not considered the role of public imbalances as an additional constraint on growth. The external imbalance considered so far in the literature includes public disequilibrium, but the impact of the latter on overall growth has not been analysed separately. The recent experience of some peripheral European countries falling into a public debt crisis is the motivation to deal with this issue. As Pelagidis and Desli (2004) argue, the implementation of an expansionary fiscal policy, aiming at strengthening growth rates and reducing unemployment, would not always achieve the desirable objectives. It could be the case that budget deficits, financed either by money printing or by public borrowing, would increase public debt and interest rates, crowd out private investments, fuel inflation, and damage medium-term growth. The issue of whether budget deficits are always desirable has many dimensions, including whether government borrowing is financing government consumption or investment in infrastructure, whether the deficit is sustainable, and how it is financed. On the other hand, the hesitation of many policy makers – especially in Europe – to rely more aggressively on fiscal policy measures in order to keep their public finances more or less balanced may lead to the possibility of a vicious cycle between low growth and higher deficit formation as a result of the reduction of tax revenues.

Our paper aims at contributing to this debate by developing an alternative growth model, in line with Thirlwall's Law, that takes into account not only external, but also internal imbalances due to budget deficits and public debt. The model also considers

that relative prices can play a significant role in the pace of economic growth. The reduced form of the growth of domestic income is determined, among other things, by factors related to fiscal policy and public finances that could affect economic growth negatively. The theoretical model is tested for the Portuguese economy that has recently been facing a serious problem with financing its public debt and thus called for external intervention. The implemented restrictive measures are expected to have negative repercussions on growth in the following years. Taking all these facts into account, the paper is organized as follows: in section 2 we develop the theoretical growth model controlling for internal and external imbalances and relative price movements; section 3 tests the model for the Portuguese economy trying to identify the main determinants of growth; a scenario analysis is provided in section 4 focusing on the factors that could foster growth in Portugal, and the last section concludes.

2. Growth model with internal and external imbalances and the role of relative prices.

Recently, Soukiazis et al. (2012) developed a multi equation model – henceforth the SCA model - to derive the reduced form of income growth which depends, among other things, on internal and external imbalances. However, for the sake of simplification the model assumed that relative prices do not play a significant role on economic growth and that in the long term international relative prices remain constant. In this paper we relax this controversial assumption. The model is in line with the balance of payments constrained growth hypothesis with three particular differences: (i) it considers not only external imbalances (current account deficits), but also internal imbalances emerging from public deficit and debt; (ii) it considers the import contents of the components of demand; (iii) relative prices are introduced into the growth model and this is the main difference from the SCA model.

2.1. Import Demand Function

We start developing the model by specifying the demand for imports equation. Contrary to the conventional specification that considers real domestic income as the main aggregate determinant of the demand for imports, we use the components of domestic income to explain import flows. Alternatively to the SCA model, we assume that relative prices do play a significant role and that in the long run they can affect

economic growth[2]. According to these assumptions, the import demand equation is specified as follows:

$$M = \alpha C^{\pi_c} G^{\pi_g} X^{\pi_x} K^{\pi_k} \left(\frac{P^* e}{P}\right)^{\delta_m} \quad \textbf{(1)}$$

where M is imports, C private consumption, G government expenditures, X exports and K investment, all expressed at constant prices. In addition, P and P^* are domestic and foreign price levels, respectively and e the prevailing exchange rate (the price of foreign currency in terms of domestic currency units). In this equation, π represents the elasticity of each of the components of demand in relation to imports. These elasticities are all expected to be positive since all components of demand have import content. In addition, δ_m is the relative price elasticity of the demand for imports with an expected negative sign.Taking logs and differentiating through time we can define the same equation in growth rates, where a lower-case letter with a dot denotes the instantaneous growth rate of a given variable:

$$\dot{m} = \pi_c \dot{c} + \pi_g \dot{g} + \pi_x \dot{x} + \pi_k \dot{k} + \delta_m (\dot{p}^* + \dot{e} - \dot{p}) \quad \textbf{(2)}$$

In this way, the growth in demand for imports $\dot{m}$ depends on the growth rates of private consumption ($\dot{c}$), government expenditures ($\dot{g}$), exports ($\dot{x}$), and investment ($\dot{k}$), respectively. Additionally, the growth of imports depends on the growth of domestic ($\dot{p}$) and foreign ($\dot{p}^*$) prices respectively, and the variation of the exchange rate ($\dot{e}$) over time. The next step is to determine the growth rates of the components of demand.

2.2. Export Demand Function

[2] The hypothesis that relative prices remain constant in the long term is a debatable assumption made in some studies for the sake of simplifying the specification of the model. As we explained before, there are studies showing that relative prices are important in international trade and explain a substantial part of growth especially in developing countries. Concerning Portugal, Garcimartín et al. (2010-11) attribute the slowdown of economic growth in Portugal to the overvaluation of the domestic currency (loss of price competitiveness) when the country joined the Euro zone.

In this function it is assumed that foreign income Y^* and relative prices of exports $(\frac{P^*e}{P})$ are the main determinants of export demand. It is explicitly assumed that exports competitiveness is based on price and non-price competitiveness captured by the price and income elasticity of the demand for exports, respectively. Therefore, we assume that relative prices change in the long-term analysis, contrary to the one price hypothesis assumed in the SCA model. Having this in mind, the export equation is defined as:

$$X = \beta Y^{*\varepsilon_x} (\frac{P^*e}{P})^{\delta_x} \quad \textbf{(3)}$$

where ε_x is the income elasticity of demand for exports capturing the non-price characteristics of the exported goods associated with quality, design, reliability, varieties, etc[3]. Additionally, δ_x is the relative price elasticity of export demand with an expected positive sign. Expressing this equation in growth rates we get:

$$\dot{x} = \varepsilon_x \dot{y}^* + \delta_x (\dot{p}^* + \dot{e} - \dot{p}) \quad \textbf{(4)}$$

where $\dot{x}$ is the growth of real exports, $\dot{y}^*$ the growth of real foreign income, $\dot{p}$ and $\dot{p}^*$ the growth of domestic and foreign prices respectively, and $\dot{e}$ the exchange rate variation.

2.3. Private final consumption

The final consumption of households is a function of total disposable income and the yields obtained by holding government bonds:

$$C = \left[((1-t) + rw_{BH})Y\right]^{\varepsilon_c} \quad \textbf{(5)}$$

[3] Although we assume that the income elasticity of demand for exports captures the quality characteristics of the produced goods we do not neglect the fact that changes in relative prices can be related to changes in relative quality as well.

where t is the tax rate on income, r is the real interest rate[4], w_{BH} is the share of home bond holders on public debt, and ε_c is the income elasticity of consumption. Taking growth rates the consumption equation becomes:

$$\dot{c} = \varepsilon_c \left(\frac{\Delta r w_{BH} + r \Delta w_{BH}}{(1-t) + r w_{BH}} + \dot{y} \right) \quad \textbf{(6)}$$

Assuming that the share of home bond holders on public debt does not change over time, $\Delta w_{BH} = 0$, the consumption function reduces to:

$$\dot{c} = \varepsilon_c \left(\frac{\Delta r w_{BH}}{(1-t) + r w_{BH}} + \dot{y} \right) \quad \textbf{(7)}$$

or alternatively

$$\dot{c} = \varepsilon_c \left(\dot{y} + \frac{(\Delta i - \Delta \dot{p}) w_{BH}}{(1-t) + r w_{BH}} \right) \quad \textbf{(8)}$$

since $\Delta r = \Delta(i - \dot{p}) = \Delta i - \Delta \dot{p}$

Therefore, consumption growth is a function of the growth of domestic income and interest rate revenues obtained by holding government bonds. In the estimation approach we will assume that consumption growth is a function of disposable income growth.

2.4. Private Investment

The main determinants of investment are after-tax income, the interest revenues from home bond holders, and the real interest rate measuring the real cost of capital borrowing:

$$K = \left[\left((1-t) + r w_{BH} \right) Y \right]^{\varepsilon_k} r^{\varepsilon_r} \quad \textbf{(9)}$$

[4] Real interest rate is the difference between nominal interest rate i and domestic inflation, $r = i - \dot{p}$.

where ε_k and ε_r are the income and interest rate elasticities with respect to changes in capital stock. Taking growth rates and following the same development as in the consumption function, the investment equation becomes:

$$\dot{k} = \varepsilon_k \left[\dot{y} + \frac{\Delta \dot{r} w_{BH}}{(1-t) + r w_{BH}} \right] + \varepsilon_r \dot{r} \qquad \textbf{(10)}$$

Substituting the change in real interest rate we get an alternative expression given by

$$\dot{k} = \varepsilon_k \left[\dot{y} + \frac{(\Delta i - \Delta \dot{p}) w_{BH}}{(1-t) + r w_{BH}} \right] + \varepsilon_r (\Delta i - \Delta \dot{p}) \qquad \textbf{(11)}$$

Therefore, the growth of capital stock is a function of the growth in income, the revenues obtained by holding governments bonds and the growth of real interest rates. In the estimation approach we will assume that growth of capital stock is determined by the growth of disposable income and the growth of real interest rates for reasons of simplification.

2.5. Government sector

We consider that the government budget is given by the following identity, in nominal terms:

$$G_n + iB_H + i^* B_F e = tYP + D \qquad \textbf{(12)}$$

where G_n is nominal government expenditures, B_H is public debt[5] owned by home bond holders, B_F is public debt owned by foreign bond holders, Y is domestic income, P is the domestic price level, D the public deficit, i and i^* are nominal interest rates paid to home and foreign public debt holders, respectively, e the nominal exchange rate, and t is the tax rate on nominal income. According to this relation, public deficit exists when total current expenditures (including interest payments on public debt) exceed the revenues obtained through taxes on domestic money income, i.e, $G_n + iB_H + i^* B_F e > tYP$.

[5] Public debt is originated by issuing government bonds to finance public deficit.

As it is shown in *Appendix A* (see Equation A.5), the long term relationship of the growth of real government expenditures $\dot{g}$ is given by:

$$\dot{g} = \frac{t\dot{y}}{w_G} + (\dot{d} - \dot{p})\frac{w_D}{w_G} - \left[\Delta i + i(\dot{b}_H - \dot{p})\right]\frac{w_{BH}}{w_G} - \left[\left(e\Delta i^* + i^*\Delta e\right) + i^* e\left(\dot{b}_F - \dot{p}\right)\right]\frac{w_{BF}}{w_G} \quad \textbf{(13)}$$

where $w_D = \frac{D}{Y}$ is the budget deficit ratio, $w_G = \frac{G}{Y}$ is the government spending ratio, $w_{BH} = \frac{B_H}{PY}$ and $w_{BF} = \frac{B_F}{PY}$ are the shares of public debt owned by home and foreign bonds holders (as a percentage of nominal income), respectively, $\dot{d}$ is the growth of budget deficit and $\dot{b}_H$ and $\dot{b}_F$ are the growth rates of the public debt owned by home and foreign bond holders, respectively.

2.6. Balance-of-payments condition

The last relation of the model is an external equilibrium condition given by the following identity:

$$XP + D_F e - i^* B_F e = MP^* e \quad \textbf{(14)}$$

The left hand side of the identity shows the money resources available to finance imports (export revenues plus the amount of public deficit assets hold by foreigners minus interest rate payments on foreign bond holders).

It is shown in *Appendix B,* Equation (B.6) that the balance of payments final relation can be expressed as:

$$\dot{x} + \dot{p} + (1-\xi)\frac{w_D}{w_X}(\dot{p} + \dot{y} - i^*) - (1-\xi)\frac{w_B}{w_X}\Delta i^* = \frac{w_M}{w_X}\frac{P^* e}{P}\left(\dot{m} + \dot{p}^* + \dot{e}\right) \quad \textbf{(15)}$$

where $\dot{x}, \dot{m}, \dot{p}, \dot{p}^*$, $\dot{y}$ and $\dot{e}$ are growth rates of exports, imports, domestic prices, foreign prices, domestic income and nominal exchange rate, respectively. Additionally, w_D, w_B, w_M and w_X are the ratios of budget deficit, public debt, imports and exports on income, respectively. Finally $(1-\xi)$ represents the percentage of public deficit (or debt) which is financed by external markets.

2.7. Domestic income growth

In Appendix C, Equation C.4 shows that the growth rate of domestic income can be predicted by the following relation:

$$\dot{y} = \frac{A}{B}$$

where

$$A = \left[\begin{array}{l} \left(\varepsilon_x - \frac{w_M}{w_X}(\frac{P^*e}{P})\pi_x\varepsilon_x \right)\dot{y}^* + \left(\delta_x(1 - \frac{P^*e}{P}\frac{w_M}{w_X}\pi_x) - \delta_m \frac{w_M}{w_X}(\frac{P^*e}{P}) \right)(\dot{p}^* + \dot{e} - \dot{p}) + \\ + \dot{p} + (1-\xi)\frac{w_D}{w_X}(\dot{p} - i^*) - (1-\xi)\frac{w_B}{w_X}i^* - \\ - \frac{P^*e}{P}\frac{w_M}{w_X}\left\{ \begin{array}{l} \frac{(\Delta i - \Delta\dot{p})\xi w_B}{(1-t) + r\xi w_B}(\pi_c\varepsilon_c + \pi_k\varepsilon_k) + \pi_k\varepsilon_r(\Delta i - \Delta\dot{p}) + \\ + \pi_g\left[-\Delta i\frac{\xi w_B}{w_G} - \Delta i^* e(1-\xi)\frac{w_B}{w_G} \right] + \dot{p}^* + \dot{e} \end{array} \right\} \end{array} \right] \quad \text{and} \quad \textbf{(16)}$$

$$B = \frac{w_M}{w_X}(\frac{P^*e}{P})\left\{ \pi_c\varepsilon_c + \pi_k\varepsilon_k + \pi_g\left(\frac{t}{w_G} + \frac{w_D}{w_G} - \frac{i\xi w_B}{w_G} - i^* e(1-\xi)\frac{w_B}{w_G} \right) \right\} -$$

$$-(1-\xi)\frac{w_D}{w_X}$$

Equation **(16)** shows that, among other factors, the growth of domestic income is determined by internal and external imbalances, taking also into account the effect of relative prices. Equation **(16)** will be used to predict actual growth in Portugal.

3. Testing the model for the Portuguese economy

The import demand equation **(2)**, the export demand equation **(4)**, the private consumption equation **(8)** and the investment equation **(11)** are estimated simultaneously to obtain the elasticities which are needed to compute the reduced form of domestic income growth as it is expressed by equation **(16)**. Annual data are used covering the period 1986-2010, thus starting at the year that Portugal joined the early

EEC. The definition of the variables and the data sources are explained in Appendix D. The method used for estimating the system equations is *3SLS (Three-Stage Least Squares)* as it is more efficient to capture the interrelation between equations and the causal and feedback effects between the variables.[6] Table E.1 in the Appendix E provides the estimation results where simultaneity is controlled by using instrumental variables. The growth of imports, consumption, investment, and exports are assumed to be endogenous as well as the growth of government expenditures, domestic disposable income, real exchange rate and real domestic interest rate. All other variables in the system are assumed exogenous, including some lagged variables, as it is explained in Table E.1.

In general the estimation results are quite satisfactory; all elasticities carry their expected signs and are statistically significant with few exceptions. The relative price elasticity is statistically significant in the import equation (at the 5% level) and carries the correct negative sign but in the export demand equation caries a wrong negative sign and it is significant at a 10% level only[7]. The value of the relative price elasticities is low in comparison with the income elasticities, confirming the general finding in the literature that trade is more sensitive to income than to price changes. The striking fact in the import demand function is the high elasticity of consumption, which exceeds unity (π_c=1.232) indicating that imports increase more than proportionally with respect to consumption increase[8]. Although the export and the investment elasticities with respect to imports are also relevant, thus indicating a significant import content in these elements of demand, they are lower (π_x=0.311 and π_k=0.414 respectively). An unexpected result is the negative government spending elasticity of imports (π_g=-0.370) and statistically significant at the 5% level. This could signify an import substitution policy of the government spending giving preference to the domestic goods and services.

Table E.1 also shows that investment and exports are income elastic with respect to domestic disposable income and foreign income, respectively (ε_k =1.85 and ε_x =2.85),

[6] For more details on the 3SLS method, see for instance, AlDakhil (1998) and Wooldridge (2002).

[7] An increase in relative prices reflects devaluation and a decrease a valuation of domestic currency. The same wrong sign of the relative prices on exports also found in previous studies for Portugal, see for instance, Soukiazis E. and Micaela Antunes (2012).

[8] However, this elasticity is not statistically different from unity.

the former confirming the accelerator principle in the investment function, and the latter showing the high sensitivity of exports relative to external demand (the OECD income growth). This high export dependence on foreign income should be a case of concern in periods of economic slowdown in foreign markets. Consumption is income inelastic, as expected (ε_c=0.822) but with a sizeable value. Finally, the impact of real interest rate is negative on investment (ε_r =-0.789), an expected result since this variable measures the real cost of financing investment projects.

We also regressed each of the equations individually, by *2SLS* (see Table E.2 in Appendix E) using the same instruments. The intention was to carry out some diagnostic tests to justify the robustness of our results. The first is the Sargan statistic, a test of over-identifying restrictions to check the validity of the instruments used in the regressions and that hypothesis is confirmed in all cases. The second is the Pagan-Hall heteroskedasticity test, showing that the hypothesis of homoskedasticity is never rejected. The third test is the Cumby-Huizinga test for autocorrelation. The null hypothesis is that errors are not first-order autocorrelated and this is confirmed in all cases. The last one is a normality test, conceptually similar to the Jarque-Bera skewness and kurtosis test. The null hypothesis is that residuals from a given regression are normally distributed, and this hypothesis is not rejected in all equations, except for imports.

Table I below reports the values which are necessary for computing the growth rates of domestic income in Portugal. Some are estimated values taken from Table E.1 (Appendix E) others are annual averages over the period considered (see Appendix D for variable definition and data sources). Three growth rates are computed: $\dot{y}_a$ obtained from equation (16) where internal and external imbalances are considered and relative prices are not neutral; $\dot{y}_b$ determined by the SCA model with relative prices being constant, and $\dot{y}_c$ obtained from Thirlwall's original Law, given by $\dot{y} = \frac{\dot{x}}{\pi}$. In the latter case, it was necessary to estimate the import demand function, $\dot{m} = \pi\dot{y} + \delta_m(\dot{p}^* + \dot{e} - \dot{p})$, by OLS (with robust standard errors) to obtain the aggregate income elasticity with respect to import growth (π=2.633).

Table I. Computation of the growth rates of domestic income. Portugal, 1986-2010

ε_x	π_x	ε_c	π_c	ε_k	π_k	π_g	ε_r	δ_m
2.854	0.311	0.822	1.232	1.850	0.414	-0.370	-0.789	-0.495
δ_x	t	r	$\dot{p}$	$\dot{y}^*$	w_D	w_G	w_B	ξ_D
-0.441	0.360	0.027	0.057	0.025	0.050	0.378	0.588	0.401
ξ_B	w_M	w_X	i	i^*	Δi	Δi^*	e	$\dot{e}$
0.401	0.364	0.280	0.084	0.055	-0.009	-0.002	0.972	0.013
$(\frac{p^* e}{p})$	$\dot{p}^* + \dot{e} - \dot{p}$	$(\Delta i - \Delta \dot{p})$	$\dot{p}^*$	$(\dot{p} - i^*)$				
1.087	-0.012	-0.001	0.026	0.002				
$\dot{y}_a$	$\dot{y}_b$	$\dot{y}_c$	$\dot{y}$					
-1.5631	-0.02%	2.335	2.728					
Internal and external imbalances and relative prices no neutral Equation (16)	Internal and external imbalances and relative prices neutral	Thirlwall's Law $\dot{y} = \frac{\dot{x}}{\pi}$	Actual growth					

Notes: $\varepsilon_x, \pi_x, \varepsilon_c, \pi_c, \varepsilon_k, \pi_k, \pi_g, \varepsilon_r, \delta_m$ and δ_x are taken from Table E.1 (see Appendix E).

r, t, w_D, w_G, w_B, w_M, w_X, i, i^*, e, $\dot{p}$ and $\dot{y}^*$ are annual averages over the period 1986-2010.

ξ_D and ξ_B = 0.401 is assumed constant over the whole period.

Comparing these different growth rates with the actual average annual growth in Portugal over the period 1986-2010 ($\dot{y}$=2.728%) the following remarks can be made:

(i) The growth rate obtained by Thirlwall's original Law ($\dot{y}_c$=2.335%) using the aggregate income elasticity of imports (π=2.633) accurately predicts actual growth rate ($\dot{y}$=2.728%) in Portugal. The Portuguese economy grew, on average, 0.393 percentage points (per annum), above the rate allowed by the balance-of-payments equilibrium. We have to recall that Thirlwall's Law is a restrictive form in the sense that it assumes that balance of payments is in equilibrium, relative prices are not playing any significant role on growth and no internal imbalances are considered in the model.

(ii) The growth rate obtained by the augmented Thirlwall's model which considers internal and external imbalances and relative prices are not neutral

($\dot{y}_a$ =-1.5631%) underestimates substantially actual growth in Portugal. This result shows that Portugal should grow much less than actually did in order not to aggravate internal and external imbalances. In other words, Portugal grew faster than the rate allowed by the balance-of-payments equilibrium and its public financial capability at the cost of accumulating internal and external deficits and this can explain the recent debt crisis of the country. In order to grow faster without deteriorating internal and external imbalances some improvements have to be made on structural parameters and especially on those related with competitiveness. We will show that more explicitly in the scenario analysis of the next section.

(iii) The fact that Portugal grew at a higher average growth rate (2.73%) than that predicted by our model in the last two decades can be explained by capital inflows financing this growth. The higher actual growth was obtained at the expense of accumulating higher external debt over time corresponding to 233% of GDP in 2009. On the other hand, the low growth rate predicted by our model (negative) is due to the fact that internal imbalances imply capital outflows via debt interest rate payments. Therefore, they play a similar role as imports, restricting growth in the long run.

(iv) Substantially different results are obtained when relative prices are considered in our model. Our previous model with no relative prices (the so called SCA model)[9] predicted a higher average growth rate, around 0.28%, in comparison to -1.5631% rate found by the new model controlling for the relative prices effects. Therefore, the difference can be attributed to the contribution of relative prices mostly affecting the import and export sectors as we have concluded from the regression analysis (see Table E.1 in Appendix E).

An important explanation for the low (negative) growth performance predicted by our model lies in the high import sensitivity of the components of demand, especially that of consumption (π_c = 1.232). This elasticity shows that if consumption increases by one percentage point (p.p.) this will induce a 1.232 p.p. increase in imports (more than

[9] See Soukiazis et al. (2012).

proportional). Therefore, a high amount of domestic consumption is spent on imported consumption goods and could be responsible for the balance of payments deficits on the current account.

The high import sensitivity of the components of demand explains the high income elasticity of the demand for imports at the aggregate level π=2.633 showing that imports grow more than twice the increase in domestic income. The high penetration of imports can also be observed by the share of imports on income, around w_M =36%, with exports representing w_X = 28% only. Therefore the multiplier effects of the components of demand on growth are not substantial in the Portuguese economy as they are counter-balanced by the increase in imports.

We have to notice here that what is important in international trade is not importing too much in order to produce domestic and exported goods, but ensuring that the transformation of imported components into domestic goods and exports contains enough value-added. In international markets, most produced goods and exports embody a substantial share of imported components, but in terms of gains it is important that the value (price) - especially of exports embodying imported components - is sufficiently higher than the value (price) of those imported components. Traditionally, Portugal produces (and exports) low value-added domestic goods (due to low productivity) despite the move from low to medium or medium-high technology exports in recent years (OECD, 2008). On the other hand, the share of the service sector in the overall economy has risen (corresponding to 75.4% of gross value added against 22.3% in industry and 2.3% in agriculture). However, labour productivity gains have been particularly weak and became negative since the beginning of the current decade. The service sector involves mainly a high number of micro enterprises (wholesale, retail, hotels and restaurants) with a substantial proportion of non-tradable goods and high informality (OECD Economic Survey, 2010).

4. A scenario analysis

Some simulations can be made with the aim to detect the factors that could help the economy to grow faster.

(i) Fiscal policy towards a reduction in income taxation. If taxation on income reduces from t = 36% to 20% (everything else constant) the predicted growth increases from $\dot{y}_a$ = -1.5631 to $\dot{y}_a$ = -1.41%. In fact there is a positive effect on growth due to a more friendly taxation policy but the stimulus is not very significant.

(ii) Budget deficit policy aiming at reducing public deficit and debt ratio. If we assume w_D = 0.03 and w_B = 0.60 (the values imposed by the Growth and Stability Pact i.e., deficit of 3% and debt of 60% of the GDP) the predicted growth is around $\dot{y}_a$ = -1.58%. Therefore public budget discipline alone does not help the economy to grow faster.

(iii) Our simulation approach shows that growth in Portugal is not sensitive to domestic interest rates changes but it is highly sensitive to changes in foreign interest rates[10]. For instance, if i^* increases by two percentage points (from the average rate of the whole period of 5.5% to 7.5%) the growth rate predicted from our model becomes much more negative, $\dot{y}_a$ = -2.91%. On the other hand if foreign interest rates are reduced by one percentage point (from 5.5% to 4.5%) the growth rate in Portugal becomes much less negative $\dot{y}_a$ = -0.93%. A combination of a 3% in budget deficit and 60% in public debt ratio (the growth and stability goals) and assuming a 2% foreign interest rate paid on government bonds raises income growth to a positive rate 0.68%. This is an interesting result showing the great difficulty Portugal is presently facing to finance its public debt. As it is known, Portugal was forced to ask the intervention of the IMF in 2011 when interest rates paid on government bonds exceeded 7%, budget deficit was around 8% and public debt around 100% of GDP. Due to this situation, austerity measures were implemented with the aim to reduce internal and external imbalances, having strong negative effects on growth and unemployment in the short term course.

(iv) The increase in the share of home government bond holders is another factor that favors growth. For instance, if half of the public debt is financed by domestic resources (ξ_B increases from 40% to 50%) our model predicts a less negative growth rate of $\dot{y}_a$ = -0.91%. A scenario of a deficit ratio of 3%, debt ratio of 60%, a foreign interest rate of 2%, and a share of 50% of home government bond holders raises income growth to a

[10] In this study we use long term interest rates of the German economy as the benchmark for foreign interest rates.

positive rate equal to $\dot{y}_a$ = 0.94%. Therefore policies to convince national savers to invest on home government bonds can enhance economic growth.

(v) The novelty in this new model is that now we assume that relative prices are not neutral. If we assume that relative prices are constant in the long run, that is, $\dot{p}^* + \dot{e} - \dot{p} = 0$, and therefore (P*e/P) =1, e=1 and $\dot{e} = 0$ and replace these values into our model (equation (16)) the obtained growth rate is $\dot{y}_b$=-0.02% which is much higher than the one found when relative prices are not neutral, $\dot{y}_a$ = -1.56% (see Table I). Therefore relative prices make a substantial difference in the growth pace and when are ignored the model over-predicts the growth rate in Portugal. The lower growth rate obtained when relative prices are included in the model can be explained by the over valuation of the domestic currency in Portugal. It is interesting to check a scenario where there is a change in the average value of the growth of real relative prices (or real exchange rate) for the whole period from -0.0116 to 0.06 representing a devaluation of domestic currency[11]. In this case it is shown that growth increases to a positive rate equal to $\dot{y}_a$ =0.15% confirming the hypothesis that a currency devaluation is a stimulus to growth increasing the country´s competiveness in foreign markets.

(vi) By reducing the import sensitivity of exports (elasticity) from π_x=0.31 (see Table I) to 0.20 our model predicts a higher growth rate equal to $\dot{y}_a$ = -0.93%. It is therefore shown that lowering the import sensitivity of exports is a stimulus to growth. When the import content of exports is high the exports' multiplier effects on income are crowded out by higher imports. Reducing the import content of exports is an appropriate policy to achieve higher growth.

(vii) Reducing the share of imports by only 4 percentage points (from 36% to 32%) the predicted growth is $\dot{y}_a$ = -1.10%, or alternatively increasing the share of exports by 4 percentage points (from 28% to 32%) the obtained growth is even higher, $\dot{y}_a$ = -0.57%. A combined policy with the aim at reducing the import share to 30% and increasing export share to 35% (having a surplus on trade) yields an even higher growth rate, around $\dot{y}_a$ = 1.30%. Therefore changing the structure of the shares of imports and exports is the appropriate way to enhance growth.

[11] This is not a pragmatic solution for Portugal since the country belongs to the eurozone and nominal exchange rates are fixed.

(viii) Finally, an alternative scenario could be to determine the equilibrium growth rate of the Portuguese economy with no internal and external imbalances and neutral relative prices. For concreteness, the following conditions are assumed: export share equal to import share w_X=w_M, therefore w_M/w_X=1; constant relative prices $\dot{p}^* + \dot{e} - \dot{p} = 0$, thus (P^*e/P) =1, e=1 and $\dot{e} = 0$; and zero deficit and debt ratio w_D=w_B=0. Replacing these values in equation (16) the equilibrium growth rate is very high $\dot{y}_a$ =5.5%. However, the above assumptions are not plausible and can over predict growth rates in Portugal.

According to these hypothetical scenarios it is clearly shown that the most effective policy to achieve higher growth in Portugal applies to the external sector, towards a balanced external trade and changing the structure of imports and exports. This is in line with Thirlwall's Law that affirms that growth is balance-of-payments constrained. Additionally, the way of financing public debt and the service payments on that debt play an important role in the growth analysis as well as a competitive devaluation.

5. Concluding remarks

The aim of this study was to develop a more complete growth model in line with Thirlwall's Law that takes into account both internal and external imbalances and assumes that relative prices are not neutral. The important contribution of the extended model is that it discriminates the import content of aggregate demand and introduces public deficit and debt measures as determinants of growth. Additionally, the model controls for relative prices movements and this is the main difference from our previous model (the SCA model). The reduced form of the model shows that growth rates can be obtained in three alternative ways: assuming internal and external imbalances and no neutrality in relative prices; assuming internal and external imbalances but neutral relative prices; and lastly the growth rate predicted by Thirlwall's Law. The growth model is tested for the Portuguese economy over the period 1986-2010 to check its accuracy.

The equations constituting the model are estimated by *3SLS* to control the endogeneity of the core variables and to obtain consistent estimates. The empirical analysis shows that growth rates obtained by Thirlwall's Law accurately predict the average growth rate

of the Portuguese economy over the period 1986-2010 although it is slightly lower than the actual growth. However, Thirlwall's Law considers some controversial assumptions, namely that external trade is balanced, public finances are at equilibrium, and relative prices are neutral. Testing our model where trade and public imbalances are allowed and relative prices are not neutral the predicted growth rate is even lower (negative) than the actual one and this is consistent with the external trade and public debt disequilibria the country has been accumulating in recent years.

The scenarios implemented to explain the low growth rate predicted by our model point to the fact that policies aiming to equilibrate external deficits or changing the structure of imports and exports are more effective for achieving higher growth. Competitive devaluation also acts as a stimulus to growth. It is also shown that policies designed to reduce public deficits and public debts, but above all to achieve better conditions of financing internal imbalances (mostly from domestic resources), and reducing the payment costs of public debt are beneficial to growth. Therefore, Portugal could benefit from the challenging idea of issuing Eurobonds to finance its public debt in the European market with lower costs.

Appendix A

Government sector

We consider that the government budget is given by the following identity:

$$G_n + iB_H + i^* B_F e = tYP + D \tag{A.1}$$

where G_n is nominal government expenditures, B_H is public debt owned by home bond holders, B_F is public debt owned by foreign bond holders, Y is domestic income, P is the domestic price level, D the public deficit, i and i^* are nominal interest rates paid to home and foreign public debt holders, respectively, e is the nominal exchange rate (the price of foreign currency in terms of national currency units), and t is the tax rate on nominal income. The above relation can be divided by P to define real government expenditures,

$$\frac{G_n}{P} + \frac{iB_H}{P} + \frac{i^* B_F e}{P} = \frac{tYP}{P} + \frac{D}{P} \qquad \text{or} \tag{A.2}$$

$$G = tY + \frac{D}{P} - \frac{iB_H}{P} - \frac{i^* B_F e}{P} \tag{A.3}$$

Taking absolute changes and dividing by G we get

$$\frac{\Delta G}{G} = \frac{t\Delta Y}{Y}\frac{Y}{G} + \Delta(\frac{D}{P})\frac{1}{G} - \Delta(\frac{iB_H}{P})\frac{1}{G} - \Delta(\frac{i^* B_F e}{P})\frac{1}{G} \tag{A.4}$$

Let's develop now the three last right hand components of equation (A.4):

$$\frac{\Delta(\frac{D}{P})}{G} = \frac{\Delta(D)P - D\Delta(P)}{P^2 G} = \frac{\Delta(D)P}{P^2 G}\frac{DY}{DY} - \frac{D\Delta(P)}{P^2 G}\frac{Y}{Y} = \dot{d}\frac{w_D}{w_G} - \dot{p}\frac{w_D}{w_G} = (\dot{d} - \dot{p})\frac{w_D}{w_G}$$

where $w_D = \frac{D}{PY}$ is the share of public deficit on income, $w_G = \frac{G}{Y}$ the government expenditure share, and $\dot{d}$ the growth of public deficit.

$$\frac{\Delta(\frac{iB_H}{P})}{G} = \Delta i \frac{B_H Y}{PGY} + i\frac{\Delta(B_H)P - B_H \Delta P}{P^2 G} = \Delta i \frac{w_{BH}}{w_G} + i\frac{\Delta(B_H)}{PG}\frac{YB_H}{YB_H} - i\frac{B_H \Delta PY}{P^2 GY} =$$

$$= \Delta i \frac{w_{BH}}{w_G} + i\dot{b}_H \frac{w_{BH}}{w_G} - i\dot{p}\frac{w_{BH}}{w_G} = \left[\Delta i + i(\dot{b}_H - \dot{p})\right]\frac{w_{BH}}{w_G}$$

where $w_{BH} = \frac{B_H}{PY}$ is the share of public debt owned by home bond holders to income and $\dot{b}_H$ is the growth of public debt owned by home bond holders.

$$\frac{\Delta(i^* \frac{B_F}{P} e)}{G} = \Delta i^* \frac{B_F eY}{PGY} + \Delta e \frac{i^* B_F Y}{PGY} + i^* e \frac{\Delta(B_F)P - B_F \Delta P}{P^2 G} = (e\Delta i^* + i^* \Delta e)\frac{w_{BF}}{w_G} +$$

$$+ i^* e\left[\frac{\Delta(B_F)P}{P^2 G}\frac{B_F Y}{B_F Y} - \frac{B_F \Delta P}{P^2 G}\frac{Y}{Y}\right] = (e\Delta i^* + i^* \Delta e)\frac{w_{BF}}{w_G} + i^* e\left(\dot{b}_F \frac{w_{BF}}{w_G} - \dot{p}\frac{w_{BF}}{w_G}\right) =$$

$$= \left[(e\Delta i^* + i^* \Delta e) + i^* e(\dot{b}_F - \dot{p})\right]\frac{w_{BF}}{w_G}$$

where $w_{BF} = \frac{B_F}{PY}$ is the share of public debt owned by foreign bond holders to real income and $\dot{b}_F$ is the growth of public debt owned by foreign bond holders.

Replacing the above expressions into equation (A.4) we obtain:

$$\dot{g} = \frac{t\dot{y}}{w_G} + (\dot{d} - \dot{p})\frac{w_D}{w_G} - \left[\Delta i + i(\dot{b}_H - \dot{p})\right]\frac{w_{BH}}{w_G} - \left[(e\Delta i^* + i^* \Delta e) + i^* e(\dot{b}_F - \dot{p})\right]\frac{w_{BF}}{w_G} \quad \text{(A.5)}$$

Equation (A.5) is the long term relation of the growth of real government spending.

Appendix B

External imbalances

The external equilibrium condition is given by

$$XP + D_F e - i^* B_F e = MP^* e \qquad \text{(B.1)}$$

Taking absolute changes and dividing by *XMP* we get

$$\frac{(\Delta X)P}{XMP} + \frac{X(\Delta P)}{XMP} + \frac{(\Delta D_F)e}{XMP} + \frac{D_F(\Delta e)}{XMP} - \frac{(\Delta i^*)B_F e}{XMP} - \frac{i^*(\Delta B_F)e}{XMP} - \frac{i^* B_F(\Delta e)}{XMP} =$$
$$= \frac{(\Delta M)P^* e}{XMP} + \frac{M(\Delta P^*)e}{XMP} + \frac{MP^*(\Delta e)}{XMP}$$

$$\frac{\dot{x}}{M} + \frac{\dot{p}}{M} + \frac{(\Delta D_F)e}{XMP} + \frac{D_F(\Delta e)}{XMP} - \frac{(\Delta i^*)B_F e}{XMP} - \frac{i^*(\Delta B_F)e}{XMP} - \frac{i^* B_F(\Delta e)}{XMP} =$$
$$= \frac{\dot{m}P^* e}{XP} + \frac{(\Delta P^*)e}{XP} + \frac{P^*(\Delta e)}{XP}$$

Multiplying the above relation by *M* we obtain

$$\dot{x} + \dot{p} + \frac{(\Delta D_F)e}{XP} + \frac{D_F(\Delta e)}{XP} - \frac{(\Delta i^*)B_F e}{XP} - \frac{i^*(\Delta B_F)e}{XP} - \frac{i^* B_F(\Delta e)}{XP} =$$
$$= \frac{\dot{m}MP^* e}{XP} + \frac{M(\Delta P^*)e}{XP} + \frac{MP^*(\Delta e)}{XP}$$

Alternatively

$$\dot{x} + \dot{p} + \frac{(\Delta D_F)e}{XP} + \frac{D_F(\Delta e)}{XP} - \frac{(\Delta i^*)B_F e}{XP} - \frac{i^*(\Delta B_F)e}{XP} - \frac{i^* B_F(\Delta e)}{XP} =$$
$$= \frac{\dot{m}MP^* e}{XP} + \frac{M(\Delta P^*)e}{XP}\frac{P^*}{P^*} + \frac{MP^*(\Delta e)}{XP}\frac{e}{e}$$

or

$$\dot{x} + \dot{p} + \frac{(\Delta D_F)}{D_F}\frac{D_F e}{XP} + \frac{D_F e}{XP}\frac{(\Delta e)}{e} - \frac{(\Delta i^*)B_F e}{XP} - \frac{i^*(\Delta B_F)}{B_F}\frac{B_F e}{XP} - \frac{i^* B_F e}{XP}\frac{(\Delta e)}{e} =$$
$$= \frac{M/Y}{X/Y}\frac{P^* e}{P}\left(\dot{m} + \dot{p}^* + \dot{e}\right) \qquad \text{(B.2)}$$

Some auxiliary relations would help to simplify equation (B.2)

We assume that total budget deficit is financed by domestic and foreign resources, as

$D = D_H + D_F e$ where $\frac{D_H}{D} = \xi_D$ is the portion of the deficit financed domestically and

$(1-\xi_D) = \frac{D_F e}{D}$ is the portion of the deficit financed from abroad. Then

$$D_F e = \frac{D_F e D}{D} = (1-\xi_D)D$$

Taking changes we obtain

$$(\Delta D_F)e + D_F(\Delta e) = (1-\xi_D)\Delta D$$

Alternatively, dividing by *D* and rearranging:

$$\frac{(\Delta D_F)e}{D}\frac{D_F}{D_F} + \frac{D_F(\Delta e)}{D}\frac{e}{e} = (1-\xi_D)\frac{\Delta D}{D} \text{ or}$$

$$\dot{d}_F(1-\xi_D) + \dot{e}(1-\xi_D) = (1-\xi_D)\dot{d}$$

dividing both sides by $(1-\xi_D)$ we get the simple relation

$$\dot{d}_F + \dot{e} = \dot{d} \qquad \text{(B.3)}$$

The same relation is valid for the public debt

$$\dot{b}_F + \dot{e} = \dot{b} \qquad \text{(B.4)}$$

since in the steady state $\xi_D = \xi_B = \xi$

Substituting these auxiliary relations in Equation (B.2) we obtain

$$\dot{x} + \dot{p} + \dot{d}_F(1-\xi)\frac{D}{XP} + \dot{e}(1-\xi)\frac{D}{XP} - B(1-\xi)\left[\frac{\Delta i^*}{XP} + \frac{i^*\dot{b}_F}{XP} + \frac{i^*\dot{e}}{XP}\right] =$$

$$= \frac{w_M}{w_X}\frac{P^* e}{P}\left(\dot{m} + \dot{p}^* + \dot{e}\right)$$

Making use of the definitions (B.3) and (B.4) the above relation reduces to

$$\dot{x} + \dot{p} + \dot{d}(1-\xi)\frac{D/YP}{(XP)/YP} - (1-\xi)\frac{B/YP}{(XP)/YP}\left[\Delta i^* + i^*\dot{b}\right] = \frac{w_M}{w_X}\frac{P^*e}{P}\left(\dot{m} + \dot{p}^* + \dot{e}\right)$$

Alternatively

$$\dot{x} + \dot{p} + \dot{d}(1-\xi)\frac{w_D}{w_X} - (1-\xi)\frac{w_B}{w_X}\left[\Delta i^* + i^*\dot{b}\right] = \frac{w_M}{w_X}\frac{P^*e}{P}\left(\dot{m} + \dot{p}^* + \dot{e}\right)$$

By definition the change in public debt is due to budget deficit, therefore

$$\dot{b} = \frac{\Delta B}{B} = \frac{D}{B} = \frac{D/YP}{B/YP} = \frac{w_D}{w_B}$$

Using this definition in the previous relation we get

$$\dot{x} + \dot{p} + \dot{d}(1-\xi)\frac{w_D}{w_X} - (1-\xi)\frac{w_B}{w_X}\Delta i^* - (1-\xi)\frac{w_B}{w_X}i^*\frac{w_D}{w_B} = \frac{w_M}{w_X}\frac{P^*e}{P}\left(\dot{m} + \dot{p}^* + \dot{e}\right)$$

Rearranging terms the above relation simplifies to

$$\dot{x} + \dot{p} + (1-\xi)\frac{w_D}{w_X}(\dot{d} - i^*) - (1-\xi)\frac{w_B}{w_X}\Delta i^* = \frac{w_M}{w_X}\frac{P^*e}{P}\left(\dot{m} + \dot{p}^* + \dot{e}\right)$$

Assuming that the budget deficit ratio w_D is constant, therefore $\frac{D}{PY}$ is constant and thus

$\dot{d} - \dot{p} - \dot{y} = 0$ or $\dot{d} = \dot{p} + \dot{y}$ (B.5)

Substituting this condition the previous equation becomes

$$\dot{x} + \dot{p} + (1-\xi)\frac{w_D}{w_X}(\dot{p} + \dot{y} - i^*) - (1-\xi)\frac{w_B}{w_X}\Delta i^* = \frac{w_M}{w_X}\frac{P^*e}{P}\left(\dot{m} + \dot{p}^* + \dot{e}\right) \qquad \text{(B.6)}$$

This is the final relation of the balance of payments condition which can be used to derive the reduced form of the growth equation in Appendix C.

Appendix C

Growth of domestic income

In order to derive the reduced form of the growth of domestic income we first substitute into Equation (B.6) the relations found for imports growth – Equation (2) in section 2.1 – and for exports growth – Equation (4) in section 2.2 – to obtain

$$\begin{aligned}&\varepsilon_x \dot{y}^* + \delta_x\left(\dot{p}^* + \dot{e} - \dot{p}\right) + \dot{p} + (1-\xi)\frac{w_D}{w_X}(\dot{p} + \dot{y} - \dot{i}^*) - (1-\xi)\frac{w_B}{w_X}\Delta i^* = \\ &= \frac{w_M}{w_X}\frac{P^* e}{P}\left[\pi_c \dot{c} + \pi_g \dot{g} + \pi_k \dot{k} + \pi_x \dot{x} + \delta_M(\dot{p}^* + \dot{e} - \dot{p}) + \dot{p}^* + \dot{e}\right]\end{aligned} \tag{C.1}$$

We substitute further in the above equation the relations found for consumption growth – Equation (8), in section 2.3 - for investment growth – Equation (11) in section 2.4 – and for government expenditure growth – Equation (13) in section 2.5, respectively:

$$\begin{aligned}&\varepsilon_x \dot{y}^* + \delta_x\left(\dot{p}^* + \dot{e} - \dot{p}\right) + \dot{p} + (1-\xi)\frac{w_D}{w_X}(\dot{p} + \dot{y} - \dot{i}^*) - (1-\xi)\frac{w_B}{w_X}\Delta i^* = \\ &= \frac{w_M}{w_X}\frac{P^* e}{P}\left\{\begin{array}{l}\pi_c\left[\varepsilon_c\left(\dot{y} + \frac{(\Delta i - \Delta\dot{p})w_{BH}}{(1-t) + r w_{BH}}\right)\right] + \pi_k\left[\varepsilon_k\left(\dot{y} + \frac{(\Delta i - \Delta\dot{p})w_{BH}}{(1-t) + r w_{BH}}\right) + \varepsilon_r(\Delta i - \Delta\dot{p}\right] + \\ + \pi_g\left[\frac{t\dot{y}}{w_G} + \left(\dot{d} - \dot{p}\right)\frac{w_D}{w_G} - \left(\Delta i + i(\dot{b}_H - \dot{p})\right)\frac{w_{BH}}{w_G} - \left(e\Delta i^* + i^*\Delta e + i^* e(\dot{b}_F - \dot{p})\right)\frac{w_{BF}}{w_G}\right] + \\ + \pi_x\left[\varepsilon_x \dot{y}^* + \delta_x\left(\dot{p}^* + \dot{e} - \dot{p}\right)\right] + \delta_M\left(\dot{p}^* + \dot{e} - \dot{p}\right) + \dot{p}^* + \dot{e}\end{array}\right\}\end{aligned} \tag{C.2}$$

The next step is to make some rearrangements with the aim to simplify relation (C.2):

(i) From Equation (B.5) we can use $\dot{d} - \dot{p} = \dot{y}$

(ii) The term $\Delta i + i(\dot{b}_H - \dot{p}) = \Delta i + i(\dot{p} + \dot{y} - \dot{p}) = \Delta i + i\dot{y}$,

since $\frac{B}{PY} = w_B$ is constant and therefore $\dot{b} = \dot{p} + \dot{y}$, implying $\dot{b}_H = \dot{p} + \dot{y}$ because

$B_H = \frac{B_H}{B} B = \xi B$. Taking changes, $\Delta B_H = \xi \Delta B$ or $\frac{\Delta B_H}{B_H} = \frac{\xi \Delta B}{B_H}$ and therefore,

$\dot{b}_H = \xi \frac{\Delta B}{B}\frac{B}{B_H} = \xi \dot{b}\frac{1}{\xi} = \dot{b}$

(iii) The term

$$\Delta i^{*} e + i^{*} \Delta e \frac{e}{e} + i^{*} e(\dot{b}_{F} - \dot{p}) = \Delta i^{*} e + i^{*} e\dot{e} + i^{*} e(\dot{b} - \dot{e} - \dot{p}) =$$
$$= \Delta i^{*} e + i^{*} e\dot{e} + i^{*} e\dot{b} - i^{*} e\dot{e} - i^{*} e\dot{p} = \Delta i^{*} e + i^{*} e(\dot{b} - \dot{p}) =$$
$$= \Delta i^{*} e + i^{*} e\dot{y}$$

since $B_F = \frac{B_F eB}{eB} = (1-\xi)\frac{B}{e}$ and therefore $\dot{b}_F = \dot{b} - \dot{e}$ and also if

$\frac{B}{PY}$ is assumed as constant in the long term, then $\dot{b} = \dot{p} + \dot{y}$ and $\dot{b} - \dot{p} = \dot{y}$

(iv) It can also be shown that $w_{BH} = \frac{B_H}{PY} = \frac{B_H}{B}\frac{B}{PY} = \xi w_B$, where ξ is the percentage of public debt financed internally, and similarly, $w_{BF} = \frac{B_F e}{PY} = \frac{B_F e}{B}\frac{B}{PY} = (1-\xi)w_B$ where (1-ξ) is the percentage of public debt financed externally.

Further substitution of the relations explained from (i) to (iv) into the above balance of payments equation and by rearranging terms, we obtain:

$$\left(\varepsilon_x - \frac{w_M}{w_X}\left(\frac{P^{*}e}{P}\right)\pi_x\varepsilon_x\right)\dot{y}^{*} + \left(\delta_x - \delta_m\frac{w_M}{w_X}\left(\frac{P^{*}e}{P}\right)\right)(\dot{p}^{*} + \dot{e} - \dot{p}) + \dot{p} + (1-\xi)\frac{w_D}{w_X}(\dot{p} - i^{*}) -$$
$$-(1-\xi)\frac{w_B}{w_X}i^{*} - \left(\frac{P^{*}e}{P}\right)\frac{w_M}{w_X}\left\{\begin{array}{l}\dfrac{(\Delta i - \Delta\dot{p})\xi w_B}{(1-t)+r\xi w_B}\left(\pi_c\varepsilon_c + \pi_k\varepsilon_k\right) + \pi_k\varepsilon_r(\Delta i - \Delta\dot{p}) + \\ +\pi_g\left(-\Delta i\dfrac{\xi w_B}{w_G} - \Delta i^{*}e(1-\xi)\dfrac{w_B}{w_G}\right) + \pi_x\delta_x(\dot{p}^{*} + \dot{e} - \dot{p}) + \dot{p}^{*} + \dot{e}\end{array}\right\} = \quad \text{(C.3)}$$
$$= \frac{w_M}{w_X}\left(\frac{P^{*}e}{P}\right)\dot{y}\left\{\pi_c\varepsilon_c + \pi_k\varepsilon_k + \pi_g\left(\frac{t}{w_G} + \frac{w_D}{w_G} - \frac{i\xi w_B}{w_G} - i^{*}e(1-\xi)\frac{w_B}{w_G}\right)\right\} - (1-\xi)\frac{w_D}{w_X}\dot{y}$$

Solving the above equation (C.3) for $\dot{y}$ we define the growth rate of domestic income which depends on a vast number of parameters and variables associated with internal and external imbalances and counting with the effect of relative prices on growth. Therefore, $\dot{y} = \frac{A}{B}$ where

$$A = \left[\begin{array}{l} \left(\varepsilon_x - \frac{w_M}{w_X}(\frac{P*e}{P})\pi_x\varepsilon_x \right)\dot{y}* + \left(\delta_x(1 - \frac{P*e}{P}\frac{w_M}{w_X}\pi_x) - \delta_m \frac{w_M}{w_X}(\frac{P*e}{P}) \right)(\dot{p}* + \dot{e} - \dot{p}) + \\ + \dot{p} + (1-\xi)\frac{w_D}{w_X}(\dot{p} - i*) - (1-\xi)\frac{w_B}{w_X} i* - \\ - \frac{P*e}{P}\frac{w_M}{w_X} \left\{ \begin{array}{l} \frac{(\Delta i - \Delta\dot{p})\xi w_B}{(1-t) + r\xi w_B}(\pi_c\varepsilon_c + \pi_k\varepsilon_k) + \pi_k\varepsilon_r(\Delta i - \Delta\dot{p}) + \\ + \pi_g \left[-\Delta i \frac{\xi w_B}{w_6} - \Delta i * e(1-\xi)\frac{w_B}{w_G} \right] + \dot{p}* + \dot{e} \end{array} \right. \end{array} \right]$$

(C.4)

$$B = \frac{w_M}{w_X}(\frac{P*e}{P}) \left\{ \pi_c\varepsilon_c + \pi_k\varepsilon_k + \pi_g \left(\frac{t}{w_G} + \frac{w_D}{w_G} - \frac{i\xi w_B}{w_G} - i*e(1-\xi)\frac{w_B}{w_G} \right) \right\} -$$

$$-(1-\xi)\frac{w_D}{w_X}$$

Equation (C.4) will be tested empirically to predict domestic growth of the economy.

Appendix D

Description of the variables and data sources

- $\dot{m}_t$ – annual growth rate of real imports - Imports of goods and services at 2000 prices (national currency; annual percentage change).
- $\dot{c}_t$ – annual growth rate of final private consumption - Private final consumption expenditure at 2000 prices (national currency; annual percentage change).
- $\dot{x}_t$ – annual growth rate of real exports - Exports of goods and services at 2000 prices (national currency; annual percentage change).
- $\dot{k}_t$ – annual growth rate of investment - Gross fixed capital formation at 2000 prices (national currency; annual percentage change).
- $\dot{y}_t$ – annual growth rate of real GDP - GDP at 2000 market prices (national currency; annual percentage change).
- $\dot{p}_t$ – annual growth rate of price deflator GDP at market prices (national currency; annual percentage change).
- $\dot{p}_t^*$ - annual growth rate of price deflator GDP at market prices, for the EU-12 (national currency; annual percentage change).
- w_G – share of government's expenditure on GDP - Total expenditure; general government minus interest including flows on swaps and FRAs (% of GDP at market prices; excessive deficit procedure).
- w_D – share of government's deficit on GDP - Net lending (-) or net borrowing (+); general government (% of GDP at market prices; excessive deficit procedure).
- w_B – share of government's debt on GDP - General government consolidated gross debt (% of GDP at market prices; excessive deficit procedure).
- w_M - imports of goods and services at current prices (national accounts) - % of GDP at market prices
- w_X-. exports of goods and services at current prices (national accounts) - % of GDP at market prices.
- t – share of government's revenues on GDP - Total current revenue; general government (% of GDP at market prices; excessive deficit procedure).
- i – nominal long-term interest rates (%)
- i^* - nominal long-term interest rates (%) for Germany

Data on $\dot{m}_t$, $\dot{c}_t$, $\dot{x}_t$, $\dot{k}_t$, $\dot{y}_t$, $\dot{p}_t$, $\dot{p}_t^*$, w_G, w_D, w_B,,w_M, w_X, t, i and i^* were taken from European Commission (2011).

- $\dot{g}_t$ – annual growth rate of government's expenditure. Computed by the authors from data on "General government expenditure by function (Millions of euro from 1.1.1999/ECU up to 31.12.1998)" (for 1990 on) and "General government expenditure (Millions of euro from 1.1.1999/ECU up to 31.12.1998)" (till 1989), from Eurostat - http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/themes (extracted on 29th February 2012 and 14th January 2011, respectively) and information on $\dot{p}_t$.
- $\dot{y}^*$ - annual growth rate of real foreign income (OECD countries). Computed by the authors using information about "Gross domestic product, GDP per head, US $, constant prices, constant PPPs, reference year 2005" , from OECD.StatExtracts - http://stats.oecd.org/ (extracted on 15th December 2011)
- $\dot{y}_d$ - annual growth rate of real disposable income. Computed by the authors using information about "Net national disposable income (national currency, constant prices,

national base year)", from OECD.StatExtracts - http://stats.oecd.org/ (extracted on 10th March 2012)

- *e* – nominal effective exchange rate - price of domestic currency in terms of foreign currency - index (2010=100) narrow indices (27 countries). Computed by the authors using monthly data, from the Bank for International Settlements(BIS)- http://www.bis.org/statistics/eer/index.htm (extracted on 18th May 2012)

- *(P*e/P)*- real effective exchange rate index (2010=100), narrow indices (27 countries). Computed by the authors using monthly data, from the Bank for International Settlements(BIS)- http://www.bis.org/statistics/eer/index.htm (extracted on 18th May 2012)

Appendix E

Table E1. The 3SLS estimation of the structural growth model: Portugal 1986-2010.

	Coefficient	Std Error	t-stat	p-value	R^2	F-stat	p-value
Imports growth							
constant	2.439	0.897	2.72	0.008***	0.9181	55.0	0.000
$\dot{c}_t$	1.232	0.348	3.53	0.001***			
$\dot{g}_t$	-0.369	0.121	-3.04	0.003***			
$\dot{x}_t$	0.311	0.075	4.17	0.000***			
$\dot{k}_t$	0.414	0.119	3.49	0.001***			
$(\dot{p}_t^* + \dot{e}_t - \dot{p}_t)$	-0.495	0.231	-2.14	0.035**			
Consumption growth							
constant	1.328	0.377	3.53	0.001***	0.6732	60.76	0.000
$\dot{y}_{d,t}$	0.822	0.105	7.79	0.000***			
Investment growth							
constant	-1.795	0.979	-1.83	0.071*	0.6943	25.73	0.000
$\dot{y}_{d,t}$	1.850	0.293	6.31	0.000***			
$\dot{r}_t$	-0.789	0.479	-1.65	0.103*			
Exports growth							
constant	-1.599	1.266	-1.26	0.210	0.6622	26.53	0.000
$\dot{y}_t^*$	2.854	0.399	7.15	0.000***			
$(\dot{p}_t^* + \dot{e}_t - \dot{p}_t)$	-0.441	0.228	-1.94	0.056*			

Table E2. The 2SLS estimation of each equation of the structural model, 1986-2010.

	Coefficient	Std Error	t-stat	p-value	Sargan test	Heteroskedasticity test	AR(1) test	Normality test
Imports growth								
constant	2.139	1.048	2.04	0.057*	χ^2_{16}=21.882	χ^2_{21}= 15.131	χ^2_1= 0.608	χ^2_2=11.17
$\dot{c}_t$	1.351	0.408	3.31	0.004**	p-value=0.1470	p-value=0.8163	p-value=0.4366	p-value= 0.0038
$\dot{g}_t$	-0.372	0.142	-2.62	0.018**				
$\dot{x}_t$	0.327	0.087	3.74	0.002***				
$\dot{k}_t$	0.355	0.139	2.55	0.021**				
$(\dot{p}_t^* + \dot{e}_t - \dot{p}_t)$	-0.489	0.271	-1.80	0.089*				
Consumption growth								
constant	1.379	0.404	3.41	0.003***	χ^2_{20}=19.49	χ^2_{21}= 18.99	χ^2_1=0.2091	χ^2_2= 2.80
$\dot{y}_{d,t}$	0.798	0.118	6.74	0.000***	p-value=0.4896	p-value=0.5855	p-value= 0.6475	p-value= 0.2468
Investment growth								
constant	-1.710	1.053	-1.62	0.120	χ^2_{19}= 22.95	χ^2_{21}=21.12	χ^2_1=0 .2752	χ^2_2= 6.77
$\dot{y}_{d,t}$	1.806	0.317	5.70	0.000***	p-value=0.2396	p-value=0.4517	p-value=0.5998	p-value= 0.034
$\dot{r}_t$	-0.942	0.535	-1.76	0.093*				
Exports growth								
constant	-1.550	1.428	-1.09	0.291	$\chi^2_{19=}$22.93	χ^2_{21}=18.97	χ^2_1=0.3871	χ^2_2= 0.43
$\dot{y}_t^*$	2.871	0.468	6.14	0.000***	p-value=0.240	p-value=0.5872	p-value= 0.5338	p-value= 0.806
$(\dot{p}_t^* + \dot{e}_t - \dot{p}_t)$	-0.378	0.274	-1.38	0.184				

Notes: Endogenous variables: $\dot{m}_t,\ \dot{c}_t, \dot{k}_t,\ \dot{x}_t,\ \dot{g}_t,\ \dot{r}_t,\ \dot{y}_{d,t}\ ,(\dot{p}_t^* + \dot{e}_t - \dot{p}_t)$

Exogenous variables: $\dot{y}_t^*, t_t, w_{B,t}, w_{d,t}, w_{g,t}, i_t, \dot{p}_t, \dot{p}_t^*, i_t^*, w_{D,t-1}, w_{G,t-1}, w_{B,t-1}, \dot{c}_{t-1}, \dot{x}_{t-1}, \dot{k}_{t-1}, \dot{g}_{t-1}, \dot{c}_{t-2}, \dot{x}_{t-2}, \dot{k}_{t-2}, \dot{g}_{t-2}, \dot{r}_{t-2}$

*, **, *** Coefficient significant at the 10%, 5% and 1% level, respectively.

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