Insider Trading and Market Efficiency
With Risk- and Ambiguity-Aversion

Paolo Vitale

University of Pescara*

November 2015†

*Department of Economics, Università Gabriele d’Annunzio, Viale Pindaro 42, 65127 Pescara (Italy); telephone: ++39-085-453-7647; fax: ++39-085-453-7565; webpage: http://www.unich.it/~vitale; e-mail: p.vitale@unich.it

†I thank Ernesto Savaglio and seminar participants at the University of Siena for valuable comments. The author alone is responsible for the views expressed in the paper and for any errors that may remain.
Insider Trading and Market Efficiency
With Risk- and Ambiguity-Aversion

ABSTRACT

We analyze the impact of risk-aversion on the trading activity of a strategic insider in an asset market with explicit trading rules. As she finds it optimal to trade more aggressively and reveal her private information at a faster pace than her risk-neutral counterpart, risk-aversion is beneficial to the efficiency of the market. Because her optimal trading strategy is identified by a min-max choice mechanism, so that in any round of trading she first identifies the worst market conditions and then selects the optimal market order against such conditions, we establish an equivalence with an alternative formulation in which an ambiguity-averse insider faces Knightian uncertain over market conditions and seeks to implement a robust trading strategy.

JEL Classification Numbers: D82, G14.

Keywords: Insider Trading, Market Efficiency, Risk-aversion, Ambiguity-aversion
Introduction

The informational efficiency of asset markets hinges on the activity of individual traders. In particular, private information on the fundamentals of stocks can only be reflected in their prices if some insiders trade such stocks to exploit their superior information. Only through the trading process asset markets may be expected to reach strong-form efficiency. Consequently, the trading strategy of privately informed agents has been extensively investigated from both an empirical and theoretical standpoint. On the theoretical front, Kyle's seminal contribution (Kyle, 1985) analyzes the optimal trading strategy of an insider in the market for a risky asset. Such insider observes one piece of fundamental information and trades over a sequence of call auctions to maximize the expected value of her total profits. Chau and Vayanos (Chau and Vayanos, 2008) extend Kyle's analysis to consider the case in which the insider repeatedly receives private signals on the asset's fundamentals.

Both Kyle and Chau and Vayanos assume that the insider is risk-neutral. However, risk-aversion is an important feature of individual preferences which heavily influences their economic choices, in particular when these have implications which reverberate over the long-run. Then, our objective is primarily that of investigating the consequence of risk-aversion on the behavior of the strategic insider considered by Chau and Vayanos and on the characteristics, such as its efficiency and liquidity, of the asset market in which she operates. A crucial issue we are interested in is whether a risk-averse insider will trade more or less aggressively, making the market more or less efficient, than her risk-neutral counterpart. Indeed, while in a static formulation a risk-averse insider ought to trade less aggressively (Subrahmanyam, 1991), this is not necessarily the case in a dynamic one (Holden and Subrahmanyam, 1994).

Our paper is related to that of Guo and Ou-Yang (Guo and Ou-Yang, 2015), who consider an infinite-horizon continuous-time model of an asset market in which a monopolistic risk-averse insider privately observes the expected growth rate of the asset's dividends. Our formulation, however, differs from theirs as we are able to analyze the insider's trading activity both in discrete-time and in the continuous-time limit, doing away with the transaction costs Guo and Ou-Yang rely on to find a meaningful equilibrium for the asset market. In addition, we abstract from their assumption that liquidity trading in the asset market is subject to mean-reverting shocks. This mean-reversion implies that not only the insider possesses superior information on the asset's fundamentals but also on the corresponding market’s liquidity conditions.

In our paper we take the opposite view, as a supplementary contribution to the current
debate on the impact of private information on the efficiency of asset markets we make is the analysis of a different formulation in which the insider is interested in maximizing the expected discounted value of her trading profits, as supposed by Chau and Vayanos, but is uncertain on the market conditions in which she will trade. Thus, we envision a corporate insider who might have access to private information on the firm’s fundamentals, but be at a disadvantage vis-à-vis dealers, brokers and other financial intermediaries which trade in the firm’s stock, as these have a better understanding of the trading activity of all market participants and consequently of the liquidity conditions which dominate the market for the firm's stock. While these trading counterparts possess a full grasp of the trading process which governs the stock price, the corporate insider may be uncertain about the nature of such process. Assuming she cannot exactly define the nature of all the forces behind the trading process and the market conditions they determine (as long as the corresponding probabilities of occurrence), the insider faces Knightian uncertainty on the stock price dynamics. Aversion to such uncertainty, namely ambiguity aversion, results in the insider adopting a robust trading strategy which serves her well in all market conditions.

Interestingly we establish that an equivalence holds between the two alternative formulations we consider. Indeed, while very different, they lead to the same conclusions pertaining to the trading activity of the insider and the characteristics of the asset market. This is because the optimal trading strategy of a risk-averse insider is identical to that of a ambiguity-averse insider who faces Knightian uncertainty, as it is identified by the same min-max choice mechanism. In this way, our contribution is also related to the literature on ambiguity in asset markets (see, among others, Dow and Ribeiro da Costa Werlang (1992); Caskey (2009); Condie and Ganguli (2011, 2014); Ozsoylev and Werner (2011); Easley, O’Hara, and Yang (2014); Mele and Sangiorigi (2014)).

This paper is organized as follows. In the next Section we describe Chau and Vayanos’ analytical framework, presenting the protocol of trading for the risky asset, the dynamics of its fundamentals and the characteristics of its market participants. Then, the preferences of the risk-averse insider are illustrated and their basic properties presented. The equilibrium concept for the sequential auction market is reformulated accordingly.

In Section 2 the new stationary linear equilibrium is characterized. The optimal trading strategy for the risk-averse insider is derived and combined with the filtering exercise of the market maker to obtain the system of equations which fully describes the stationary equilibrium. Then, the asymptotic behavior of the stationary equilibrium when the interval of time
between consecutive auctions approaches zero is analyzed. As the sequential auction market approaches its continuous time analogous explicit solutions to the system of equations describing the stationary equilibrium exist. These allow to derive clear-cut implications on the impact of risk-aversion on the trading activity of the insider and the efficiency and liquidity of the asset market.

In Section 3 we discuss the impact of risk-aversion on the insider’s trading activity and market quality when the frequency of trading is finite. The conclusions drawn for the continuous-time limit are shown to be preserved in the discrete-time formulation. Thus, the impact of risk-aversion on the insider’s trading and the efficiency of the asset market is shown to be substantial. Regulatory implications for the accessibility of the asset market to traders and automated trading are also derived. Specifically, a more risk-averse insider will trade more aggressively, revealing a larger proportion of her private information and increasing the efficiency of the market.

Furthermore, a comparative static exercise indicates that, differently from what holds for Chau and Vayanos’ original formulation, the volume of liquidity trading in the asset market and the degree of informational advantage of the insider affect her trading intensity, the informative signal conveyed by her orders and the efficiency of the market. A similar conclusion is drawn when increasing the frequency of trading. As this augments the insider’s trading activity becomes more informative and the asset market more efficient, an effect which is magnified by her risk-aversion. This suggests that recent developments in securities markets, notably the advent of high frequency trading, should not impair market quality as it might be feared.

In Section 4 a different formulation, in which the insider is concerned with her discounted profits but is uncertain about the market conditions in which she operates, is considered. Because of the Knightian nature of her uncertainty, she employs a robust trading strategy which works well in all market conditions. Such trading strategy is defined and shown to correspond to the optimal trading strategy of the risk-averse insider introduced in Section 1. This establishes the equivalence between the two alternative formulations we consider. A final Section proposes some concluding remarks.
1 Sequential Auctions, Insider Trading and Risk-Aversion

In Chau and Vayanos (2008) a monopolistic insider privately observes signals on the growth rate of dividends paid out by a risky asset. She trades with a competitive market maker according to a protocol of trading which corresponds to Kyle's (Kyle, 1985) sequential auction market. In their formulation private information pertains to the dividend growth rate and not to the liquidation value of the risky asset. In addition, there is no terminal date for the liquidation of the risky asset and hence it is possible to consider a stationary market equilibrium in which in any round of trading the insider maximizes the expected value of her discounted future profits, while the competitive market maker breaks even by fixing the transaction price for the risky asset equal to the expected discounted value of all future dividends it pays out. While richer and more realistic than Kyle's original Chau and Vayanos' formulation maintain his assumption that both the insider and the market makers are risk-neutral.

In the sequential auction market described by Kyle, the insider cannot anticipate the transaction price at which she is going to trade the risky asset as this is conditioned by random liquidity market orders, i.e. market orders submitted by a population of liquidity or noise traders the insider can neither observe nor predict. In Kyle's original formulation, as the volume of liquidity trading varies, so does the liquidity of the asset market and the expected profits of the insider. Market efficiency is however unaffected, as the insider finds it optimal to preserve the noise-to-signal ratio in the flow of orders observed by the market maker. Such feature of Kyle's formulation hinges on the assumption that the insider is risk-neutral. If risk-averse, she would be concerned with the variability of her payoffs, which is conditioned by the volume of liquidity trading. As such volume increases so does the variability of the insider's payoffs and the amount of risk she bears.

As shown by Holden and Subrahmanyam (1994), with a risk-averse insider the properties of the asset market in Kyle's sequential equilibrium change. In particular, the dependence of the market characteristics, such as its liquidity and efficiency, on the volume of liquidity trading, the amount of informational asymmetry and the degree of the insider's risk-aversion is much richer. Importantly, they show that as the degree of risk-aversion increases so does the efficiency of the market.

A related issue is then whether risk-aversion exerts similar effects on the insider's trading strategy and on market quality within Chau and Vayanos' richer formulation. Characterizing the market equilibrium under risk-aversion is a challenging exercise. This is because, while
under risk-neutrality the insider solves a standard linear-quadratic-Gaussian (LQG) optimal control problem, this is no longer the case when she is risk-averse. Thus, in Holden and Subrahmanyam’s analysis the risk-averse insider solves a linear-exponential-quadratic-Gaussian (LEQG) optimal control problem. In addition, while in Kyle’s original formulation there is no discounting of future payoffs, in Chau and Vayanos’ formulation discounting future dividends is a crucial condition to be satisfied in order to derive a meaningful equilibrium. Unfortunately, as shown by Bouakiz and Sobel (1984), discounting cannot be easily accommodated within the linear-exponential-quadratic-Gaussian framework used by Holden and Subrahmanyam, as this would not permit a stationary equilibrium. In what follows we will show how to combine properly time-discounting and risk-aversion within Chau and Vayanos’ formulation and how to characterize the new market equilibrium. This is achieved by introducing a recursive optimization criterion a l`a Hansen and Sargent (Hansen and Sargent, 1994, 1995) which allows to separate the contribution of time-discounting and risk-aversion on the insider’s preferences.

1.1 The Analytical Framework

In Chau and Vayanos’ set up trading of a risky asset takes place at equally-spaced-in-time call auctions. At time \( t = n \cdot \Delta \), where \( \Delta \) represents a time interval and \( n \) is an integer number, a call auction for the risky asset is called by a market maker. He acts competitively, in that Bertrand competition with other marker makers will force him to break even. In the generic auction \( n \) market orders are submitted by the market maker’s clients, as in Kyle’s original call auction market. These orders are batched together and passed to the market maker, who then sets a unique transaction price at which all market orders are executed. This transaction price is equal to the expected fundamental value of the risky asset given the information possessed by the market maker in auction \( n \).

Clients comprise a population of liquidity traders, who are unsophisticated agents which trade for liquidity reasons and place unpredictable market orders uncorrelated with the fundamentals of the risky asset. More precisely, the overall market order of the population of liquidity traders in \( n \) is given by a white noise process, with \( \epsilon_n^l \sim N(0, \sigma^2_l \Delta) \). Beside this group of unsophisticated traders, an informed agent trades for speculative reason. This insider acts strategically, as she takes into account the impact of her orders on the transaction price, and possesses some private information on the profitability of the risky asset.

\[ \text{See also Whittle (1990).} \]
The risky asset pays in period \( n \) a dividend equal to \( d_n \Delta \). The dividend yield, \( d_n \), reverts to a time-varying value, \( g_n \), according to the following Markovian specification,

\[
d_n = d_{n-1} + \nu \Delta (g_{n-1} - d_{n-1}) + \epsilon_d^n, \tag{1.1}
\]

where \( \nu > 0 \) with \( \nu \Delta \in (0,1) \), while \( \{\epsilon_d^n\} \) are i.i.d., normally distributed shocks with mean zero and variance \( \sigma_d^2 \Delta \). In addition, the asset’s underlying profitability, \( g_n \), reverts to a long-run mean value \( \bar{g} \), according to another Markovian specification

\[
g_n = g_{n-1} + \kappa \Delta (\bar{g} - g_{n-1}) + \epsilon_g^n, \tag{1.2}
\]

where \( \kappa > 0 \) with \( \kappa \Delta \in (0,1) \), while \( \{\epsilon_g^n\} \) are i.i.d., normally distributed shocks with mean zero and variance \( \sigma_g^2 \Delta \), independent of the shocks \( \{\epsilon_d^n\} \). While convoluted these assumptions on the process generating the asset’s dividends render this analytical framework rich and realistic, as it is found that usually corporate dividends follow a dynamics consistent with an AR(2) process.

In period \( n \) firstly the market maker’s clients submit their market orders; secondly, the dividend yield, \( d_n \), is publicly observed and the insider privately observes the risky asset’s underlying profitability, \( g_n \); thirdly, the market maker selects the transaction price, \( p_n \), for the risky asset at which all orders are executed.

As the risky asset is traded for a risk-free bond which pays a continuously compounding interest rate \( r \), so that its gross return over period \( n \) is \( \exp(r \Delta) \), the transaction price chosen by the competitive market maker in auction \( n \) according to the semi-strong form efficiency condition is

\[
p_n = E \left[ \sum_{j=n}^{\infty} e^{-r(j-n)\Delta} d_j \Delta \mid \Omega_m^m \right], \tag{1.3}
\]

where \( \Omega_m^m \) denotes the market maker’s information set in period \( n \) which contains the history of aggregate order flow, \( \{x_n + \epsilon_l^n, x_{n-1} + \epsilon_l^{n-1}, x_{n-2} + \epsilon_l^{n-2}, \ldots\} \), and of the dividend yields, \( \{d_n, d_{n-1}, d_{n-2}, \ldots\} \). It can be shown that this transaction price is equal to

\[
p_n = A_0 d_n + A_1 \hat{g}_n + A_2 \bar{g}, \tag{1.4}
\]

where \( \hat{g}_n = E \left[ g_n \mid \Omega_m^m \right], A_0 = \frac{\Delta}{1-e^{-(r+\mu)\Delta}}, A_1 = A_0 \frac{(1-e^{-\nu\Delta})e^{-r\Delta}}{1-e^{-(r+\kappa)\Delta}} \) and \( A_2 = A_0 \frac{(1-e^{-\nu\Delta})(1-e^{-\kappa\Delta})e^{-2r\Delta}}{(1-e^{-r\Delta})(1-e^{-(r+\kappa)\Delta})} \), with \( \mu \) such that \( e^{-\mu\Delta} = 1 - \nu \Delta \). Since the insider observes the history of the risky asset's underlying profitability her valuation of its fundamental value, \( v_n \), is equal to the present value.
of the expected dividends given her information. It can be shown that the fundamental value
$v_n$ respects a formulation similar to equation (1.4) with $g_n$ replacing $\hat{g}_n$,

$$v_n = A_0d_n + A_1g_n + A_2\bar{g}.$$  (1.5)

Chau and Vayanos assume the insider is risk-neutral and therefore maximizes the expected
present value of the profits her trading activity generates. In period $n$ these are given by
$$\pi_n = x_n(v_n - p_n),$$
while the present value of her profits is $$\Pi_n = \sum_{j=n}^{\infty} e^{-r(j-n)\Delta} \pi_j.$$ In
period $n$ the insider chooses the market order which maximizes $E[\Pi_n | \Omega^i_n]$, where $\Omega^i_n$ denotes
her information set. This contains the history of past dividends, $\{d_{n-1}, d_{n-2}, \ldots\}$, transaction
prices, $\{p_{n-1}, p_{n-2}, \ldots\}$ and of the underlying profitability of the risky asset, $\{g_{n-1}, g_{n-2}, \ldots\}$.

Under these assumptions, if the trading strategy of the insider in period $n$ is described by a
simple linear function of the perceived error in the market maker’s estimate of the underlying
profitability, $g_{n-1} - \hat{g}_{n-1}$, so that $x_n = \beta(g_{n-1} - \hat{g}_{n-1})$, the market maker’s expectation of $g_n$
evolves according to the following expression

$$\hat{g}_n = \bar{g} + (1 - \kappa \Delta)(\hat{g}_{n-1} - \bar{g}) + \lambda_d(d_n - (1 - \nu \Delta)d_{n-1} - \nu \Delta \hat{g}_{n-1}) + \lambda_x (x_n + \epsilon_n)^{(1.6)}$$

with

$$\lambda_d = \frac{(1 - \kappa \Delta) \Sigma_g \sigma^2 \Delta}{\Sigma_g (\beta^2 \sigma^2_d + \nu^2 \sigma^2_l \Delta^2) + \sigma^2_d \sigma^2_l \Delta},$$  (1.7)

$$\lambda_x = \frac{(1 - \kappa \Delta) \beta \Sigma_g \sigma^2_l}{\Sigma_g (\beta^2 \sigma^2_d + \nu^2 \sigma^2_l \Delta^2) + \sigma^2_d \sigma^2_l \Delta},$$  (1.8)

and $\Sigma_g$ the conditional variance of $g_n$ given the market maker’s information at the end of
period $n$.

1.2 A Risk-averse Insider

To introduce risk-aversion one might consider a CARA utility of the insider’s discounted future
profits, such as $u(\Pi_n) \equiv -\exp(-\rho \Pi_n)$. In this case the insider would solve a LEQG optimal
control problem where time-separable costs are discounted. However, Bouakiz and Sobel
(1984) show that in LEQG problems with time-discounting the optimal decision rules fail to
be time-invariant, while the effects of risk-aversion wear off and the decision rules eventually
converge to what would prevail in the usual LQG problems. Consequently, if the insider were
to maximize the expected value of the CARA utility of her discounted future profits, the opti-
mal trading strategy she would choose would entail that the impact of risk-aversion dissipates over time, eventually converging to the risk-neutral counterpart.$^2$

To avoid such unpleasant features, we rely on the optimization criterion à la Hansen and Sargent (Hansen and Sargent, 1994, 1995) provided by Vitale (Vitale, 2013) which discriminates between time-discounting and risk-aversion in the formulation of individual preferences. Specifically, because profits are time-separable and the dynamics of the transaction price is Markovian, we assume that in any auction $n$ the insider chooses her optimal market order, $x_n$, solving the following recursive optimization$^3$

$$
\mathcal{V}_n = \min_{x_n} \left\{ \frac{2}{\rho} \ln \left( E_n \left[ \exp \left( \frac{\rho}{2} (c_n + e^{-r\Delta} \mathcal{V}_{n+1}) \right) \right] \right) \right\}, \quad (1.9)
$$

where $\rho$ (with $\rho > 0$) is the coefficient of risk-aversion, $e^{-r\Delta}$ is the per-period discounting factor, $c_n$ is a per-period cost function equal to the opposite of her per-auction profits, $-\pi_n = (p_n - v_n)x_n$, and $\mathcal{V}_n$ is the optimization criterion in $n$.

The optimization criterion in (1.9) accommodates risk-aversion through the curvature of the exponential function,$^4$ while the coefficient $e^{-r\Delta}$, which pre-multiplies the next period optimization criterion $\mathcal{V}_{n+1}$, captures discounting from period $n+1$ to period $n$. The optimization criterion represents a particular formulation of Epstein-Zin preferences and hence it inherits most of their properties.$^5$ In particular, differently from standard time-separable preferences, in the optimization criterion (1.9) the inter-temporal rate of substitution and the coefficient of relative risk-aversion are unrelated, so that time-discounting and risk-aversion have separate effects on the insider’s trading activity.

For $\rho \downarrow 0$, the recursive optimization in (1.9) converges to $\mathcal{V}_n = \min_{x_n} E_n[c_n + e^{-r\Delta} \mathcal{V}_{n+1}]$. This corresponds to the Bellman equation which solves the insider’s optimization exercise within Chau and Vayanos’ formulation. This implies that using the optimization criterion in (1.9) allows a genuine extension of their analysis.

Given the optimization criterion in (1.9), the strategic insider solves an optimal control

\begin{itemize}
  \item $^2$See also Whittle (1990) and Hansen and Sargent (1994).
  \item $^3$The optimization criterion proposed by Hansen and Sargent (Hansen and Sargent, 1994, 1995) differs from the formulation presented in equation (1.9) because in theirs the per-period cost $c_n$ is deterministic and hence outside the expectation operator, while in ours is stochastic. If in Chau and Vayanos’ formulation the insider could anticipate the exact value of her trading profits in auction $n$, the two criteria would coincide.
  \item $^4$The functional form $\ln \left( E \left[ \exp \left( \frac{\rho}{2} X \right) \right] \right)$ is monotonic increasing and convex in $X$, where $X \equiv c_n + e^{-r\Delta} \mathcal{V}_{n+1}$. As the convexity of $\ln \left( E \left[ \exp \left( \frac{\rho}{2} X \right) \right] \right)$ increases with $\rho$, this coefficient determines the insider’s degree of risk-aversion.
  \item $^5$See Tallarini (Tallarini, 2000) and Vitale (Vitale, 2013) for extensive discussions of the properties of the optimization criterion in (1.9) and of its relation to Epstein-Zin preferences.
\end{itemize}
problem characterized by a clear trade-off. In fact, a larger market order today generates larger profits now at the expense of future ones, since a more informative order is passed to the market maker reducing his uncertainty on the underlying profitability of the risky asset. On the other hand, the market maker solves a filtering problem, as he uses the signal contained in the flow of orders and in the dividend stream to up-date his expectation of such profitability.

To solve simultaneously and consistently these two problems we rely on the notion of sequential equilibrium introduced by Kyle. We adapt it to the scenario in which the insider recursively solves (1.9). Firstly, we define the strategies that characterize the insider and the market maker's choices. These are two collections of functions, $X$ and $P$, that indicate the trading strategy of the insider and the pricing rule of the market maker for any auction $n$,

$$X = \langle X_{-\infty}, \ldots, X_1, \ldots, X_n, \ldots, X_{\infty} \rangle, \quad P = \langle P_{-\infty}, \ldots, P_1, \ldots, P_n, \ldots, P_{\infty} \rangle. \quad (1.10)$$

Given these strategies, in any auction $n$ the insider’s market order and the transaction price will be respectively $x_n = X_n(\Omega^i_n)$ and $p_n = P_n(\Omega^m_n)$.

Secondly, we define a modified notion of sequential auction equilibrium which takes into account the insider’s revised preferences.

**Definition 1** A sequential auction equilibrium is a pair $(X^*, P^*)$ such that in any auction:

1. the insider chooses her market order by solving the recursive optimization in (1.9);
2. the market maker sets the transaction price according to the efficiency condition (1.3).

Before we proceed to characterize the equilibrium it is worth noticing that as $p_n - v_n = A_1(\hat{g}_n - g_n)$, we can introduce the modified optimization criterion $U_n = V_n / A_1$ (where $0 < A_1 < 1$). Then, it is immediate to see that the recursive optimization (1.9) corresponds to

$$U_n = \min_{x_n} \left\{ \frac{2}{A_1\rho} \ln \left( E_n \left[ \exp \left( \frac{A_1\rho}{2} \left( c^*_n + e^{-r \Delta U_{n+1}} \right) \right) \right] \right) \right\}, \quad (1.11)$$

where $c^*_n = -(g_n - \hat{g}_n)x_n$. This implies that we can simply set $A_1 = 1$, as this is equivalent to rescaling the risk-aversion coefficient to $\rho^* = \rho A_1$. Such a simplification is useful in that: i) we will simplify the algebra without any loss of generality; and ii) given that Chau and Vayanos also employ this normalization, we will have a characterization of the equilibrium which collapses to theirs for $\rho \downarrow 0$. 

9
2 The Linear Equilibrium

We will concentrate on stationary linear equilibria. To characterize them let us consider first the insider’s trading strategy, assuming that the market maker sets the transaction price of the risky asset according to equation (1.4), where his conditional expectation of the asset’s underlying profitability is given by equation (1.6). This implies that

\[ g_n - \hat{g}_n = (1 - (\kappa + \lambda_d \nu)\Delta)(g_{n-1} - \hat{g}_{n-1}) - \lambda x n + \epsilon_n, \]  

(2.1)

where \( \epsilon_n \equiv -\lambda_x \epsilon_n^l - \lambda_d \epsilon_n^d + \epsilon_n^g \) is a combination of the liquidity shock, \( \epsilon_n^l \), and of the innovations to the dividend yield, \( \epsilon_n^d \), and the asset’s underlying profitability, \( \epsilon_n^g \). Its variance is \( \sigma^2_r = (\lambda_x^2 \sigma^2_I + \lambda_d^2 \sigma^2_d + \sigma^2_g)\Delta \). Then, we can prove the following Lemma.

**Lemma 1** Assume that the market maker sets the transaction price according to equation (1.4) and formulates his expectations of the risky asset’s underlying profitability according to equation (1.6). Then, the insider’s optimal stationary trading strategy is such that in any auction:

1) her optimal market order is found solving the double recursion

\[ -B(g_{n-1} - \hat{g}_{n-1})^2 = \min_{x_n} \max_{\epsilon_n} \left[ c_n - e^{-r\Delta} B(g_n - \hat{g}_n)^2 - \frac{1}{\rho \sigma^2_r} \epsilon_n^2 \right]; \]  

(2.2)

2) the optimization criterion is a quadratic form in \( g_{n-1} - \hat{g}_{n-1} \),

\[ \mathcal{V}_n = -B(g_{n-1} - \hat{g}_{n-1})^2 - C, \]  

(2.3)

with \( B \) and \( C \) positive constants.

**Proof.** See the Appendix.

Lemma 1 posits that our risk-averse insider selects as her optimal trading strategy a min-max strategy. Thus, her market order in auction \( n \) is derived by: i) identifying the shock \( \epsilon_n \) that maximizes the argument in the brackets of equation (2.2); and ii) selecting the order \( x_n \) which minimizes such argument vis-a-vis such worst-case shock. To appreciate the economic meaning of such result, consider that the argument in the square brackets of equation (2.2) contains three terms: the per-period cost, \( c_n \); a quadratic term in \( g_n - \hat{g}_n \) which corresponds to the future value of the optimization criterion; and a penalization term, \( -\frac{1}{\rho \sigma^2_r} \epsilon_n^2 \), which captures
the impact of the insider’s uncertainty on her utility. While under risk-neutrality she would simply minimize with respect to $x_n$ the sum of the first two terms, when risk-averse she adjusts her trading to the impact of the worst-case shock on her utility. In this way the insider acts as if she were pessimistic, considering this worst-case shock very likely, applying what, following Whittle’s (Whittle, 1990) terminology, can be called a pessimistic choice mechanism. Exploiting Lemma 1 we can now prove the following Lemma, which describes the exact specification of the insider’s market order in any auction $n$ according to her optimal stationary strategy, alongside the exact formulation for the corresponding optimization criterion $V_n$.

**Lemma 2** Suppose in any auction $n$ the market maker sets the transaction price of the risky asset according to equation (1.4), where his conditional expectation of the asset’s underlying profitability is given by equation (1.6). According to the insider’s optimal trading strategy in any auction $n$ her market order is

$$x_n = \beta(g_{n-1} - \hat{g}_{n-1}), \text{ with}$$

$$\beta = \frac{(1 - (\kappa + \nu\lambda_d)\Delta)(1 - 2e^{-r\Delta}\lambda_xB)}{2\lambda_x(1 - e^{-r\Delta}\lambda_xB) + \frac{1}{2}\rho\sigma^2},$$

while the optimization criterion is as in equation (2.3), with

$$B = \frac{e^{r\Delta}}{2\lambda_x} \left(1 + \frac{1}{4\lambda_x} \rho\sigma^2 \right) \left[ \left(1 + \frac{1}{4\lambda_x} \rho\sigma^2 \right)^2 - e^{-r\Delta}(1 - (\kappa + \nu\lambda_d)\Delta)^2 \right]^{1/2} \text{ and } (2.6)$$

$$C = \frac{1}{1 - e^{-r\Delta}} \frac{1}{\rho} \ln(1 + e^{-r\Delta} \rho\sigma^2 B).$$

**Proof.** See the Appendix.

Corollary of Lemma 2 is the following result which confirms that our formulation subsumes that of Chau and Vayanos.

**Corollary 1** For $\rho \downarrow 0$ the insider’s optimal trading strategy converges to that of the risk-neutral insider derived by Chau and Vayanos.

Turning to the market maker’s filtering exercise, if the insider chooses her market order according to equation (2.4), with $\beta$ some positive constant, from the projection’s theorem for Normal distributions we see that the market maker’s expectation of $g_n$ evolves according to
equation (1.6) with \( \lambda_d \) and \( \lambda_x \) as in equations (1.7) and (1.8). The market maker’s conditional variance for \( g_n \) at the end of auction \( n \), given his information set \( \Omega_m^n \), must be in steady state a time-invariant value \( \Sigma_g \). This can be shown to be equal to

\[
\Sigma_g = \frac{(1 - \kappa \Delta)^2 \sigma_d^2 \sigma_l^2 \Delta}{\Sigma_g (\beta^2 \sigma_d^2 + \nu^2 \sigma_l^2 \Delta^2) + \sigma_d^2 \sigma_l^2 \Delta} \Sigma_g + \sigma_g^2 \Delta. \tag{2.8}
\]

Combining all results presented in this Section, we see that in equilibrium the transaction price for the risky asset set by the market maker in \( n \) is a linear function of the dividend yield, \( d_n \), the market maker’s conditional expectation of the asset’s underlying profitability, \( \hat{g}_n \), and its long-run mean value, \( \bar{g} \). As in \( n \) the market maker receives informative signals on this profitability from the dividend yield, \( d_n \), and his clients’ market orders, \( x_n + \epsilon_n \), his conditional expectation of \( g_n \) is a linear function of such variables. At the same time, the insider exploits her informational advantage by submitting a market order which is linear function of the perceived mis-pricing of the risky asset, measured by the difference between the actual profitability and the corresponding market maker’s conditional expectation, \( g_{n-1} - \hat{g}_{n-1} \).

The following Proposition sums up this characterization of a stationary linear equilibrium.

**Proposition 1** In a stationary linear equilibrium in auction \( n \) the market maker sets the transaction price for the risky asset according to equation (1.4), where the conditional expectation for its underlying profitability, \( g_n \), is a linear function of the dividend yield and his clients’ overall market order as given in equation (1.6), with the coefficients \( \lambda_d \) and \( \lambda_x \) described in equations (1.7) and (1.8), and the corresponding conditional variance is given in equation (2.8). The insider’s market order is a linear function of the market maker’s mis-pricing of the asset as given in equation (2.4), with the coefficient \( \beta \) given in equation (2.5), while her optimization criterion is given by equation (2.3), with the coefficients \( B \) and \( C \) given in equations (2.6) and (2.7).

Such equilibrium exists if there exist values for the coefficients \( B \), \( C \), \( \beta \), \( \lambda_d \), \( \lambda_x \) and \( \Sigma_g \) which simultaneously respect the equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8). In the Appendix we discuss which condition must be met for this system of equations to present a unique solution.

While explicit formulae for the coefficients \( \beta \), \( \lambda_d \), \( \lambda_x \), \( \Sigma_g \), \( B \) and \( C \) which identify the stationary linear equilibrium are not available, a simple numerical procedure allows to solve the system of equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8). This is a numerical procedure
which yields the trading intensity implicit in any initial guess $\beta_0$, $\beta = \mathcal{N}(\beta_0)$. In particular, starting from an initial guess $\beta_0$, the conditional variance $\Sigma_g$ is derived from equation (2.8); then, the coefficient $\lambda_d$, defining the price impact of the dividend yield, and the liquidity coefficient $\lambda_x$, which determines the price impact of order flow, are obtained from equations (1.7) and (1.8); eventually, equations (2.6) and (2.7) yield the constants $B$ and $C$, while equation (2.5) allows to obtain a final value for the trading intensity $\beta$.

A root of the numerical procedure $\mathcal{N}(\cdot)$ yields a solution to the system of equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8). Finding such root is simplified by the fact that, as shown next, an explicit and unique solution for the system of equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8) always exists for the continuous-time limit. Then, one can start the procedure searching for the root of $\mathcal{N}(\cdot)$ from the value of $\beta$ consistent with the stationary linear equilibrium which prevails in the continuous-time limit.

### 2.1 The Continuous-time Limit

For $\Delta$, the time interval between two consecutive auctions, converging to 0 the continuous-time limit of the linear equilibrium described in Section 2 can be considered. When $\Delta \downarrow 0$ trading approaches a continuous auction, as traders can trade the risky asset at any time. Then, the following Proposition holds.

**Proposition 2** In the continuous-time limit a unique stationary linear equilibrium for the asset market exists. The stationary linear equilibrium illustrated in Proposition 1 is characterized in the continuous-time limit by the following asymptotic behavior of the coefficients which describe it

\[
\lim_{\Delta \downarrow 0} \frac{\beta}{\sqrt{\Delta}} = \left(2\kappa + r + \rho \sigma_g \sigma_l\right)^{1/2} \frac{\sigma_l}{\sigma_g},
\]

(2.9)

\[
\lim_{\Delta \downarrow 0} \frac{\Sigma_g}{\sqrt{\Delta}} = \frac{1}{\left(2\kappa + r + \rho \sigma_g \sigma_l\right)^{1/2}} \frac{\sigma_g^2}{\sigma_l^2},
\]

(2.10)

\[
\lim_{\Delta \downarrow 0} \frac{\lambda_d}{\sqrt{\Delta}} = \frac{\nu}{\left(2\kappa + r + \rho \sigma_g \sigma_l\right)^{1/2}} \frac{\sigma_g^2}{\sigma_l^2},
\]

(2.11)

\[
\lim_{\Delta \downarrow 0} \frac{\lambda_x}{\sqrt{\Delta}} = \frac{\sigma_g^2}{\sigma_l},
\]

(2.12)
\[
\lim_{\Delta \downarrow 0} B = \frac{1}{2} \frac{\sigma_l}{\sigma_g}, \\
\lim_{\Delta \downarrow 0} C = \frac{1}{r} \sigma_g \sigma_l.
\]

**Proof.** See the Appendix.

By continuity Proposition 2 suggests that for \( \Delta \) small enough the system of equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8) does have a solution. In addition, inspection of the asymptotic behavior of the coefficients describing the stationary linear equilibrium of Proposition 1 proves the following Corollary, which illustrates some important implications for the impact of risk-aversion on the insider’s trading activity and the asset market’s performance.

**Corollary 2** In the continuous-time limit, a risk-averse insider will trade more aggressively than her risk-neutral counterpart, increasing the speed at which private information is impounded into the asset’s price and benefiting the market efficiency.

Indeed, we immediately see that for \( \Delta \downarrow 0 \) the limit of \( \beta / \sqrt{\Delta} \) is larger for \( \rho > 0 \) than for \( \rho = 0 \), while that of \( \Sigma_g / \sqrt{\Delta} \) is smaller. On the contrary, the limit of \( \lambda_x \) is independent of \( \rho \), while that of \( \lambda_d / \sqrt{\Delta} \) is smaller for \( \rho > 0 \). This indicates that in the continuous-time limit a risk-averse insider finds it optimal to trade more aggressively than her risk-neutral counterpart, choosing a larger trading intensity for the market orders she submits and revealing to the market maker a larger proportion of her private information. Consequently, the market is more efficient and the market maker learns more from order flow and less from the dividend yield on the underlying profitability of the risky asset with a risk-averse insider than with a risk-neutral one.

Another interesting Corollary of Proposition 2 is the following.

**Corollary 3** In the continuous-time limit, the market liquidity, as measured by the market depth \( 1 / \lambda_x \), is unaffected by the insider’s degree of risk-aversion.

*Prima facie* this result may appear to contradict the asymptotic behavior of the insider’s trading intensity. In fact, in the limit the ratio \( \beta / \sqrt{\Delta} \) takes a larger value when the insider is more risk-averse. Then, one wonders how the asset market can be equally liquid with continuous trading for different values of \( \rho \). Indeed, as the insider trades more aggressively and places larger market orders when risk-averse, *ceteris paribus* adverse selection should induce
the market maker to reduce market liquidity. However, $\Sigma_g$ is also smaller and hence the market maker’s uncertainty on the fundamental value of the risky asset is attenuated. Corollary 3 indicates that in the continuous-time limit these two contrasting effects on the liquidity coefficient $\lambda_x$ (that positive of a larger $\beta$ and that negative of a smaller $\Sigma_g$) exactly compensate each other, so that the market liquidity is unaffected by the insider’s degree of risk-aversion.

A third Corollary of Proposition 2 is the following.

Corollary 4 In the continuous-time limit, the effects of risk-aversion and time-discounting on the trading activity of the insider, the efficiency and liquidity of market are similar but distinct.

This is because $\rho$ and $r$ enter additively in the expressions for the limit behavior of $\beta$, $\Sigma_g$, while neither appears in the expression for the limit behavior of $\lambda_x$. As time-discounting and risk-aversion enter separately into the recursive optimization criterion (1.9) they have distinct effects on the insider’s trading activity and on market quality. Such effects are however similar in that, as shown in the next Section when discussing the discrete-time formulation, they both favor early resolution of uncertainty on the part of the insider.

Proposition 2 allows to derive interesting comparative static results. Thus, in the continuous-time limit the impact of risk-aversion on market efficiency is larger the larger the volume of liquidity trading, measured by $\sigma_l^2$. As already mentioned, when the insider is risk-neutral the volume of liquidity trading does not affect the efficiency of the market and the speed at which the insider reveals her private information on the underlying profitability of the risky asset to the market maker. On the contrary, a risk-averse insider will choose to trade more aggressively when $\sigma_l^2$ is larger. This is because a risk-averse insider cares for the variability of her payoffs and, ceteris paribus, a larger volume of liquidity trading increases such variability. The insider will then choose to reveal a larger proportion of her private information as in this way she manages to offset the negative impact of a larger $\sigma_l^2$ on the variability of her payoffs.

An additional comparative static result pertains to the percentage of information on the underlying profitability of the risky asset which is revealed to the market maker. This is given by $1 - \Sigma_g/\sigma_g^2$. In the continuous-time limit, when the insider is risk-neutral this percentage is independent of the unconditional variance of the underlying profitability, $\sigma_g^2$, which measures the degree of informational asymmetry prevailing in the asset market. On the contrary, when the insider is risk-averse such percentage is clearly increasing with $\sigma_g^2$. This is because when adverse selection is more severe the variability of her payoffs is more pronounced. Consequently, the insider is induced to trade more aggressively revealing a larger proportion of her
It is important to establish whether the conclusions drawn for the continuous-time limit are also valid when trading takes place at equally-spaced-in-time call auctions. As mentioned, the system of equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8) for \( \Delta > 0 \) does not have an explicit solution and a numerical procedure is called for. In what follows the numerical procedure illustrated in Section 2 is used for a benchmark parametric configuration proposed by Chau and Vayanos. In particular, in their calibration they employ data for Coca-cola stock. For this stock estimated values for the volatility of dividends, \( \sigma_d \), and of the underlying profitability, \( \sigma_g \), are respectively 1.06 and 0.62. The estimated values for the mean-reverting coefficients for the processes governing the dividend yield, \( \nu \), and underlying profitability, \( k \), are respectively 1.47 and 0. The continuously compounding interest rate is 2 percent.

In their calibration the standard deviation of liquidity trading, \( \sigma_l \), is normalized to 1, as this parameter does not influence the optimal trading strategy of the insider and the efficiency of the market. Proposition 2 indicates that such irrelevance result does not survive the introduction of risk-aversion, as the volume of liquidity trading affects the speed at which a risk-averse insider reveals her private information. For easy of comparison we will maintain their benchmark choice for \( \sigma_l \) but we will also discuss what happens when we modify it. Finally, we experiment with different values for the risk-aversion coefficient, \( \rho \), ranging from 0 to 1, and with the time interval between subsequent auctions, \( \Delta \). For \( \Delta \) we choose values ranging from 1/252, for daily trading, to 1/120960, for minute-by-minute trading.

[ Figure 1 about here. ]

In Figure 1 we propose plots of the dependence of the equilibrium coefficients \( \beta, \Sigma_g, \lambda_x \) and \( \lambda_d \) on \( \Delta \) for the benchmark choice of the parameters. The coefficients \( \beta, \Sigma_g, \lambda_x \) and \( \lambda_d \) which solve the system of equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8) are plotted for a frequency of the auctions varying from the daily to the minute-by-minute one. These coefficients are compared to their continuous-time limits as dictated by Proposition 2. Thus, in the top-left, top-right and bottom-right panels the ratios \( \Sigma_g/\sqrt{\Delta}, \beta/\sqrt{\Delta} \) and \( \lambda_d/\sqrt{\Delta} \) are plotted against \( 1/\Delta \), while in the bottom-left panel the coefficient \( \lambda_x \) is plotted against \( 1/\Delta \).
Comparing the actual behavior of these coefficients to their asymptotic counterparts we see that convergence to the continuous-time limit is achieved fairly rapidly. This is particularly evident for the liquidity coefficient $\lambda_x$, which already at the daily frequency is less than 10 percent away from its asymptotic value. Interestingly, this plot also suggests that market liquidity decreases with the frequency of the auctions. Therefore, an implication of our analysis is that technological innovations which increase the pace of the market do not necessarily make securities markets more liquid. Such finding can be of particular interest for regulators’ attitude toward high frequency trading and in general for the debate among regulators, practitioners and researchers on its impact on liquidity, efficiency and other features of market quality.

[Figure 2 about here.]

From Proposition 2 we concluded that in the continuous-time limit a risk-averse insider will trade more aggressively, revealing a larger proportion of her private information and increasing the efficiency of the asset market. In Figure 2 we plot the dependence of the equilibrium values of the coefficients $\Sigma_g$ (top-left panel), $\beta$ (bottom-left panel), $\lambda_x$ (top-right panel) and $\lambda_d$ (bottom-right panel) on $\rho$, for the benchmark parametric constellation and for four different values of $\Delta$, corresponding to the weekly, daily, hourly and minute-by-minute frequency of trading. The dependence of the four coefficients on $\rho$ is clear-cut and consistent with the implications of Proposition 2 for the continuous-time limit.

In particular, for all four choices of $\Delta$, the more risk-averse the insider the larger the trading intensity ($\beta$ is larger) of her market orders. As with a larger $\rho$ more information on the underlying profitability of the risky asset is conveyed by the insider’s market orders, the market maker gives more weight to order flow ($\lambda_x$ is larger) and less to the stream of dividends ($\lambda_d$ is smaller) in updating his expectations of $g_n$. Since with a larger $\rho$ more information on the underlying profitability is elicited from order flow, the market maker is less uncertain on the underlying profitability of the risky asset, so that the conditional variance of $g_n$ given the information he extracts from order flow and the stream of dividends, $\Sigma_g$, is smaller. Consequently, the asset market is more efficient and less liquid.

As the frequency of trading rises, the volume of liquidity trading observed in a single auction falls. The insider is then forced to trade less aggressively ($\beta$ is smaller). Nevertheless, the asymptotic behavior of $\beta$ indicates that $\beta/\sqrt{\Delta}$ converges to a constant for $\Delta \downarrow 0$. This implies that, as the number of auctions in one year is $1/\Delta$, the overall volume of trading by the insider in a given spell of time augments when the trading frequency rises. Consequently, as the
frequency of trading increases, order flow becomes more informative and the asset market becomes more efficient ($\Sigma_g$ is smaller) and less liquid ($\lambda_x$ is larger). This suggests that the recent advent of high frequency trading in securities markets, while reducing their liquidity should actually increase their efficiency. Once again, such conclusions may be relevant to the recent debate on the impact of high frequency trading in securities markets.

Quantitatively, the impact of risk-aversion is substantial, in particular when considering a large trading frequency. Thus, for hourly trading we see that the conditional variance $\Sigma_g$ more than halves for $\rho$ varying from 0 to 1. The percentage drop is even more pronounced for minute-by-minute trading.

[ Figure 3 about here. ]

From Proposition 2 we concluded that in the continuous-time limit a risk-averse insider will trade more aggressively when the volume of liquidity trading is larger, revealing an even larger proportion of her private information to the marker maker. In Figure 3 we plot the dependence of the coefficients $\Sigma_g$ (top-left panel), $\beta$ (bottom-left panel), $\lambda_x$ (bottom-middle panel) and $\lambda_d$ (bottom-right panel) on $\rho$ for two different values of $\sigma_l^2$ and for two different values of $\Delta$.

Consistently with Proposition 2 we see that the market maker’s conditional variance of the underlying profitability of the asset risk, $\Sigma_g$, is smaller when the volume of liquidity trading is larger. Indeed, as the volume of liquidity trading augments the insider trades more aggressively for all values of $\rho$, since her market orders are more easily disguised among those of the liquidity traders. However, when $\rho > 0$ the increase in her trading intensity is so pronounced to the point that she finds it optimal to reveal a larger proportion of her private information. This confirms our claim that the irrelevance of the liquidity conditions on the efficiency of the asset market established by Chau and Vayanos hinges on the assumption of risk-neutrality. Our result applies to the continuous-time limit as well as to the formulation with a finite trading frequency.

In Figure 3 we also plot the half-life of the market maker’s prediction error of the risky asset’s fundamental value ($t_{0.5}$; top, middle panel) as well as the half-life of such prediction error relative to the benchmark scenario with no insider trading ($t_{0.5}/t_{0.5,0}$; top, right panel). If in auction $n$ the market maker’s prediction error is $v_n - p_n$, $t_{0.5}$ indicates the time it must elapse (while $t_{0.5}/\Delta$ is the number of auctions which must be run) before such value is expected to halve, i.e. $t_{0.5}$ is such that $E_n[v_{n+t_{0.5}/\Delta} - p_{n+t_{0.5}/\Delta}] = \frac{1}{2}(v_n - p_n)$, while $t_{0.5,0}$ is the corresponding value in absence of insider trading. This half-life measures the speed of convergence of
the market maker's forecast to the fundamental value and can be considered an alternative measure of market efficiency which indicates the actual speed with which private information is diffused in the market.

This half-life is fairly high (close to one year) when no insider operates into the asset market and it is much smaller when an insider enters it. In addition, consistently with results in Figure 2, we see that the larger the frequency of trading, the smaller this half-life. Indeed, as the insider’s trading intensity, $\beta$, is of order $\Delta^{1/2}$ the information content of order flow (more precisely, the signal-to-noise ratio in order flow) augments with the trading frequency. Also in line with results unveiled by Figure 2, we see that the half-line dramatically falls when $\rho$ augments. Thus, for the hourly frequency and the large volume of liquidity trading the half line is 0.0469 (i.e. about 12 days) when $\rho = 0$ and it is 0.0163 (i.e. about 4 days) when $\rho = 1$. In brief, Figure 3 proposes even more compelling evidence of how important the attitude of the insider towards risk is in determining the efficiency of the asset market.

The results outlined in Figures 2 and 3 and in Proposition 2 depend on the trade-off between current and future profits the insider faces in choosing her market orders. In fact, a larger order generates larger profits now at the expense of smaller future ones, given that more information is passed to the market maker reducing his uncertainty on the underlying profitability of the risky asset. In Kyle’s and Chau and Vayanos’ formulations the solutions of this trade-off are very different. In Kyle’s formulation the insider finds it optimal to limit the information content of order flow, while in Chau and Vayanos’ the insider trades very quickly consuming a larger proportion of her informational advantage.

This difference is due to three features of their formulation that are absent in Kyle’s: i) the insider discounts future profits; ii) the market maker observes over time a public signal, the dividend stream, that conveys information on the underlying profitability of the risky asset; and iii) the insider’s private information becomes obsolete over time due to the mean reverting in the process governing the asset’s underlying profitability. All these features induce the insider to be more impatient in Chau and Vayanos’ formulation. Both when $\Delta > 0$, as suggested by Figures 2 and 3, and in the continuous-time limit, as shown by Proposition 2, for $\rho > 0$ the insider becomes even more impatient, this effect of risk-aversion on the timing of her trading being particularly pronounced when the volume of liquidity trading is large.

As mentioned, a larger market order by the insider affects her profit opportunities. However, it does not condition only the expected value of her profits but also their variance. While the variability of her profits is inconsequential for a risk-neutral insider, it is crucial for a risk-averse
one. The latter is actually willing to forgo part of her expected profits to reduce their variability. This means that she will be willing to trade more aggressively than it would be optimal under risk-neutrality, revealing more information to the market maker than under risk-neutrality, as this will make the prices at which she will trade less volatile and her profits less uncertain. Since the variability of such profits is larger the larger the volume of liquidity trading, for a larger $\sigma_l^2$ the impact on risk-aversion on the insider’s trading strategy will be stronger.

To explain this result one should consider that the optimization criterion in (1.9) is a special formulation of Epstein and Zin (Epstein and Zin, 1989) preferences where the coefficient of relative risk-aversion is larger than the inverse of the inter-temporal elasticity of substitution (Tallarini, 2000). Exploiting results from Kreps and Porteus (Kreps and Porteus, 1978), Epstein and Zin show that under such condition their preferences induce earlier resolution of uncertainty vis-à-vis the case of expected utility. Therefore, when $\rho$ is positive the insider is willing to accept smaller expected profits to reduce their variability. Notice, that this also clarifies why risk-aversion and time-discounting have similar effects on the insider’s activity and on market quality as unveiled by Corollary 4. In fact, with a smaller time-discounting factor, $\exp(-r\Delta)$, the insider assigns less weight in her trading choice to future payoffs. Then, a larger $r$ will also induce her to trade more aggressively, revealing a larger proportion of her private information and increasing market efficiency.

This explanation of the results outlined in Figures 2 and 3 and in Proposition 2 is confirmed by Figure 4. Here, we plot the market maker’s conditional ($E_n[V_n]$; left panel) and unconditional ($E[V_n]$; right panel) expectation of the insider’s optimization criterion, $V_n$. We notice that the more risk-averse the insider, the larger these expected values are. The minimum is reached when the insider is risk-neutral, as she maximizes the expected value of her discounted profits. When risk-averse she accepts a smaller value for her profits so that her expected optimization criterion, which according to the definition in (1.9) is minimized, is larger.

The two plots presented in Figure 4 also indicate that as the trading frequency augments the conditional and unconditional expectation of $V_n$ falls for any value of $\rho$, showing that the insider is better able to deal with the trade-off between the expected value and the variability of her profits her trading decisions entail. This occurs because as $\Delta$ decreases her uncertainty on her profits reduces. In fact, her profits in $n$ are proportional to $g_n - \hat{g}_n$, so that their variance,
given the information she possesses before submitting her market order, is increasing with that of the excess profitability \( g_n - \hat{g}_n \). From equation (2.1) we see that the conditional variance of \( g_n - \hat{g}_n \) is equal to \( \sigma_i^2 \), which is decreasing in \( \Delta \).

As for \( \Delta \downarrow 0 \), \( \sigma_i^2 \) vanishes, the uncertainty of the risk-averse insider over her profits dissipates when the frequency of trading rises to infinite and hence her optimization criterion, \( V_n \), collapses to that of her risk-neutral counterpart, as also shown by Proposition 2. It is then unsurprising that in the limit the market maker's conditional expectation \( E_n[V_n] \) converges to \( -\frac{1}{t}\sigma_g \sigma_i \), i.e. the opposite of the conditional expectation of the discounted profits of the risk-neutral insider.

### 4 Risk-aversion versus Ambiguity-aversion

There is an alternative interpretation of the stationary linear equilibrium described in Section 2. In particular, we may consider a different scenario, one in which the insider is uncertain about the liquidity conditions which characterize the market for the risky asset. It is in fact quite possible that the insider possesses a full grasp of the asset’s fundamentals, but knows very little about the liquidity conditions prevailing in the corresponding market. In particular, she may be uncertain on the trading activity of the other market maker’s clients. Consequently she could be at a disadvantage vis-à-vis the market maker, as the latter possesses a better understanding of the functioning of the market for the risky asset. In general, she might fear that she does not fully understand all the market forces which condition the asset price.

To formalize the notion that the insider is uncertain on such market conditions and on other forces not related to the asset’s fundamentals which drive its price, we make the following assumptions. Firstly, the insider supposes the asset price set by the market maker in auction \( n \) is primarily perturbed by the overall market order submitted by the population of liquidity traders. She assumes that in auction \( n \) this market order is normally distributed with mean 0 and variance \( \sigma_l^2 \Delta \), i.e. \( \epsilon_l \sim N(0, \sigma_l^2 \Delta) \). Secondly, she fears that her assumption may be incorrect and that other unspecified forces may actually influence the asset price in period \( n \). This implies that, while the insider assumes that the following model specification (corresponding to equation (2.1))

\[
g_n - \hat{g}_n = (1 - (\kappa + \lambda d)(\nu)) (g_{n-1} - \hat{g}_{n-1}) - \lambda x x_n + \epsilon_n
\]
approximates the dynamics of the excess profitability of the risky asset (or equivalently, for $A_1$ normalized to 1, that of the market maker’s mis-pricing of the asset, $v_n - p_n$), she suspects that in fact this is given by the following \textit{distorted} specification

$$g_n - \hat{g}_n = (1 - (\kappa + \lambda \nu)\Delta) (g_{n-1} - \hat{g}_{n-1}) - \lambda x_n + \epsilon_n + \sigma_x w_n,$$

(4.1)

with $w_n$ a generic function of economic history $H_n \equiv h(g_{n-1} - \hat{g}_{n-1}, g_{n-2} - \hat{g}_{n-2}, \ldots)$ which captures all unspecified forces that condition the asset price. Given the indeterminacy of $w_n$ in the definition of the distorted specification (4.1) the uncertainty of the insider on the dynamics of the excess profitability corresponds to \textit{Knightian uncertainty}. In fact, as the insider cannot determine the exact probabilities of the outcomes any trading choice will entail, she cannot measure exactly the risk she is facing. Following, Hansen and Sargent (Hansen and Sargent, 2008), we can represent the insider’s Knightian uncertainty relying on the notion of \textit{discounted conditional entropy} employed in the literature on \textit{robustness}.

To define this we introduce a measure of probabilistic discrepancy between the approximating and distorted specifications in auction $n$. In particular, introduce the state variable $z_n \equiv g_n - \hat{g}_n$ and let $f_a(z_n \mid z_{n-1})$ and $f_d(z_n \mid z_{n-1})$ be the probability density function of $z_n$, conditional on $z_{n-1}$, respectively under the approximating and distorted specifications. Denote with $m(f_a(z_n \mid z_{n-1}))$ the ratio between the log density functions, $\log(f_a(z_n \mid z_{n-1})) / \log(f_d(z_n \mid z_{n-1}))$. The \textit{conditional relative entropy} in $n$ is then defined as the conditional expectation of the log-likelihood ratio for the approximating and the distorted specifications, calculated under the distorted one,

$$I(f_a, f_d)(z_n) \equiv \int m(f_a(z_n \mid z_{n-1})) f_d(z_n \mid z_{n-1}) dz_n.$$

Hansen and Sargent show that, because under the two specifications $z_n$ is normally distributed, this conditional relative entropy is equal to $\frac{1}{2} w_n^2$. An inter-temporal measure of probabilistic distance between the two specifications is then given by the expected discounted value of all conditional relative entropies, or \textit{discounted conditional entropy},

$$R_n \equiv 2 E_n \left[ \sum_{j=0}^{\infty} e^{-r(j+1)\Delta} I(f_a, f_d)(z_{n+j}) \right] = E_n \left[ \sum_{j=0}^{\infty} e^{-r(j+1)\Delta} w_{n+j}^2 \right],$$

where the expectation is taken in $n$ under the distorted specification. This aggregate measure represents the probabilistic distance between the approximating specification assumed by the insider and the distorted one she suspects is regulating the dynamics of $g_n - \hat{g}_n$. We assume that the insider will consider as potential alternatives all the distorted specifications for which
\( R_n < \phi \), where \( \phi \) (with \( \phi > 0 \)) defines the maximum probabilistic distance between the approximating and distorted specifications she assumes feasible. Such value indicates the degree of Knightian uncertainty the insider is facing.

In this way one envisions a situation in which the insider assumes that \( g_n - \hat{g}_n \) is generated according to equation (2.1) and suspects that it is actually governed by equation (4.1). This distorted specification is assumed to be not too far from the approximating one. In measuring their distance the insider refers to the discounted conditional entropy \( R_n \). A robust trading strategy is then one which works for all distorted specifications for which \( R_n \leq \phi \). The selection criterion which defines a robust trading strategy for the insider is particularly demanding, in that it requires that in any auction \( n \) she chooses the market order which minimizes the expected aggregate cost of the worst distorted specifications (among all admissible ones). Formally, recalling that \( c_n = -\pi_n \) (i.e. the opposite of the insider’s trading profits), she selects a robust trading strategy solving in \( n \) the following constrained program,

\[
\min_{\{x_{n+j}\}_{j=0}^\infty} \max_{\{w_{n+j}\}_{j=0}^\infty} E_n \left[ \sum_{j=0}^\infty e^{-r_j \Delta_t} c_{n+j} \right],
\]

s.t. \( g_n - \hat{g}_n \) respects equation (4.1) and \( R_n \leq \phi \).

This means that first among all alternative specifications the insider chooses the worst-one, i.e. the one which minimizes the present value of her discounted profits, and second she selects the optimal market order which maximizes such profits within this worst-case scenario.\(^6\)

The particularly restrictive selection criterion set out in program (4.2) allows the insider to deal with her inability to calculate exactly the probability of the outcomes of her trading activity. More importantly, it conforms to the preferences of an agent who is averse to ambiguity and hence permits to represent the aversion of the insider to the Knightian uncertainty she faces in the asset market. In fact, Maccheroni, Marinacci, and Rustichini (2006) define a large class of preferences characterized by aversion to Knightian uncertainty, according to which an ambiguity-averse agent will choose her optimal actions solving program (4.2). Under this class of preferences the coefficient \( \phi \) measures the agent’s degree of ambiguity-aversion. This means that in the alternative formulation presented here, the larger \( \phi \) the more averse to Knightian

\(^6\)This formulation does not allow the insider to learn over time about the actual dynamics of the state variable, \( z_n \), and the pricing process. Learning could mitigate and in the long-run eliminate the insider’s uncertainty over the pricing process, as suggested by Hansen and Sargent (2007). However, if the degree of Knightian uncertainty is limited (\( \phi \) is small) it is plausible to assume that the insider finds it impossible to learn the correct pricing process.
uncertainty the insider is.⁷

As shown by Hansen and Sargent the constrained program (4.2) is equivalent to the following multiplier program

\[
\min \{ x_n + \sum_{j=0}^{\infty} e^{-r\Delta} (c_{n+j} - e^{-r\Delta} \vartheta w_{n+j}^2) \},
\]

(4.3)

s.t. \( g_n - \hat{g}_n \) respects equation (4.1) and \( \vartheta \) is some positive constant.

The constant \( \vartheta \) is shown to depend negatively on \( \phi \) \( (\vartheta = \vartheta(\phi) \text{ decreasing in } \phi) \), so that a smaller value for \( \vartheta \) corresponds to a greater degree of ambiguity-aversion. Given the value function in \( n + 1 - B(g_n - \hat{g}_n)^2 - C \), this program admits the Bellman equation

\[
-B(g_{n-1} - \hat{g}_{n-1}) - C = \min_{x_n} \max_{w_n} E_n [c_n - e^{-r\Delta} \vartheta w_n^2 - e^{-r\Delta} B(g_n - \hat{g}_n)^2 - e^{-r\Delta} C],
\]

where the expectation is taken with respect to the distribution of \( \epsilon_n \). Crucially, this Bellman equation yields the same modified Riccati equation and the same optimal market order of the non-stochastic version in which \( \epsilon_n \equiv 0 \). In the non-stochastic version of the Bellman equation \( C \) disappears, so that we find that

\[
-B(g_{n-1} - \hat{g}_{n-1})^2 = \min_{x_n} \max_{w_n} E_n [c_n - e^{-r\Delta} \vartheta w_n^2 - e^{-r\Delta} B(g_n - \hat{g}_n)^2] \text{ with }
\]

\[
g_n - \hat{g}_n = (1 - (\kappa + \lambda_d \nu) \Delta) (g_{n-1} - \hat{g}_{n-1}) - \lambda_x x_n + \sigma_e w_n. \tag{4.4}
\]

Since the Bellman equation (4.4) corresponds to the double recursion in equation (2.2) for \( \epsilon_n \) replaced by \( \sigma_x w_n \) and \( 1/\rho \) replaced by \( e^{-r\Delta} \vartheta \), the following Lemma, which posits that when the market maker sets the asset price according to the linear pricing rule presented in Section 1 the optimal stationary trading strategy of an ambiguity-averse insider coincides with that of a particular risk-averse insider, is immediately proved.

**Lemma 3** Suppose that in auction \( n \) the market maker sets the transaction price of the risky asset according to equation (1.4), where his conditional expectation of the asset’s underlying profitability is given by equation (1.6), and assumes the insider selects her market order according to the robust program (4.2). For any value of the ambiguity-aversion coefficient \( \phi \) there is a corresponding value for the risk-aversion coefficient \( \rho \) such that Lemma 2 describes the optimal

---

⁷Alternative, closely related, axiomatizations of ambiguity-aversion have been put forward by Gilboa and Schmeidler (1989), Schmeidler (1989), Epstein (1999) and Ghirardato and Marinacci (2002). For a general presentation of the literature on ambiguity-aversion see Machina and Siniscalchi (2013).
trading strategy of the ambiguity-averse insider.

Indeed, Lemma 1 shows that the optimal stationary trading strategy of our risk-averse insider is identified via a pessimistic choice mechanism, according to which in any auction $n$ the insider first isolates the shock $\epsilon_n$, which most penalizes her and then chooses as her market order that which represents the best hedge against such shock. As such choice mechanism is analogous to that which yields the optimal trading strategy according to the robust selection criterion proposed by Hansen and Sargent, Lemma 3 is established.

If the market maker is aware of the insider’s concerns and knows that they are misplaced he will still set the transaction price of the risky asset according to equation (1.4), with his conditional expectation of the asset’s underlying profitability given by equation (1.6). This establishes that when the insider faces Knightian uncertainty on the liquidity conditions governing the asset market and is ambiguity-averse, the following Proposition holds.

**Proposition 3** With Knightian uncertainty on the part of an ambiguity-averse insider, a stationary linear equilibrium is characterized by a pricing rule for the market maker and a trading strategy for the insider which coincide with those described in Proposition 1. Thus, equations (1.4) and (2.4), with the coefficients $\beta$, $\lambda_d$, $\lambda_x$, $B$ and $\Sigma_g$ respecting equations (2.5), (1.7), (1.8), (2.6) and (2.8) and $\rho$ replaced by $e^{r \Delta \varpi} / \varpi$, describe the insider’s optimal market order and the market maker’s transaction price in any auction $n$.

In brief, we conclude that our analysis of the market equilibrium described in Section 3 also applies to the realistic scenario in which a trader possesses some private information on the fundamentals of a risky asset but at the same time is uncertain about some other forces which move the asset price. As she cannot determine the exact probabilities of the profit outcomes her trades will generate, the insider will select a robust trading strategy which work well in all market conditions. Interestingly, given the results outlined in Section 3, our analysis suggests that amid an uncertain environment an informed trader will be more rather than less aggressive, revealing her private information at a faster pace and increasing the efficiency of the market in which she operates.

These conclusions contrast with the portfolio inertia typically exhibited in models of asset markets with ambiguity-averse investors (Dow and Ribeiro da Costa Werlang, 1992; Caskey, 2009; Condie and Ganguli, 2011, 2014; Ozsoylev and Werner, 2011; Easley, O’Hara, and Yang, 2014; Mele and Sangiorgi, 2014). In these models ambiguity-averse individuals fail to trade if
prices are not sufficiently favorable as to overcome their Knightian uncertainty on their payoffs. As these traders are less aggressive vis-à-vis their expected-utility-maximizer counterparts, then ambiguity-aversion reduces price informativeness and market efficiency.

Such disparity stems from three important facets of our analysis which are novel with respect to the existing literature on ambiguity in asset markets. In particular, according to our formulation: i) uncertainty pertains to liquidity conditions and not to fundamentals or to signals received on such fundamentals; ii) the ambiguity-averse trader acts strategically; and iii) she solves a dynamic rather than a static optimization exercise, so that she considers the future implications of her trading decisions. Indeed, we argue that an interesting parallel exists between the impact of risk- and ambiguity-aversion on the performance of asset markets. In fact, similarly to what occurs for a risk-averse trader, a more ambiguity-averse insider is less aggressive when the horizon of her trading activity is short and is more aggressive when such horizon is long.

Concluding Remarks

We have extended the analysis of Chau and Vayanos of an asset market, in which an insider possesses some private information on the asset’s underlying profitability and trades strategically to exploit such information, to consider the impact of risk-aversion and Knightian uncertainty on the optimal trading strategy of the insider and the market characteristics.

We have found that risk-aversion has dramatic implications on the trading activity of the insider and on the efficiency and liquidity of the market. As risk-aversion favors early resolution of uncertainty, a risk-averse insider will trade more aggressively than her risk-neutral counterpart, revealing a larger proportion of her informational advantage and increasing the efficiency of the asset market.

Our analysis also offers some normative implications, as we argue that institutional features of asset markets, such as the access of unsophisticated traders to the trading venue and the regulation of high frequency trading, conditions heavily the trading activity of the insider. Indeed, the impact of risk-aversion on the insider’s trading activity and the efficiency of the market is more pronounced when the volume of liquidity trading in the asset market is larger, as this makes the environment in which the risk-averse insider operates riskier. This means that any rule which facilitates the access of unsophisticated traders to the asset market will
actually make it more efficient.

Finally, we have proved that an equivalence exists between a formulation with a risk-averse insider and an alternative one in which the insider faces Knightian uncertainty on the market conditions which dominate the asset market and is averse to ambiguity. Such equivalence is relevant to all situations in which a trader in possession of some private information on the fundamentals of an asset does not fully comprehend its pricing process.

References


Appendix

• Proof of Lemma 1.
To prove this Lemma we first need to establish a preliminary result.

Lemma 4 If $Q(x, \epsilon)$ is a quadratic form in the vectors $x$ and $\epsilon$ which admits the saddle point value $\max_x \min_\epsilon Q(x, \epsilon)$, then the following holds

$$\min_x \int \exp \left[ -\frac{1}{2} Q(x, \epsilon) \right] d\epsilon \propto \exp \left[ -\frac{1}{2} \max_x \min_\epsilon Q(x, \epsilon) \right].$$

Proof. Consider the quadratic form $Q(x, \epsilon)$ in the vectors $x$ and $\epsilon$, where

$$Q(x, \epsilon) = \begin{pmatrix} x \\ \epsilon \end{pmatrix}' \begin{pmatrix} Q_{xx} & Q_{x\epsilon} \\ Q_{\epsilon x} & Q_{\epsilon\epsilon} \end{pmatrix} \begin{pmatrix} x \\ \epsilon \end{pmatrix}.$$ 

Assume $Q$ admits a minimum in $\epsilon$ in that $Q_{\epsilon\epsilon}$ is positive definite. Then, the following holds

$$\int \exp \left[ -\frac{1}{2} Q(x, \epsilon) \right] d\epsilon \propto \exp \left[ -\frac{1}{2} \min_\epsilon Q(x, \epsilon) \right].$$

(A.1)

In fact, for $\hat{\epsilon}$ the vector $\epsilon$ minimizing $Q$, we can write $Q(x, \epsilon) = Q(x, \hat{\epsilon}) + (\epsilon - \hat{\epsilon})'Q_{\epsilon\epsilon}(\epsilon - \hat{\epsilon})$. Consider that as $Q_{\epsilon\epsilon}$ is positive definite and invertible, the minimum of $Q$ with respect to $\epsilon$ is obtained for
\[ \dot{\epsilon} = -Q_{\epsilon\epsilon}^{-1} Q_{\epsilon x} x \] and is equal to \( Q(x, \dot{\epsilon}) = x'(Q_{xx} - Q_{x\epsilon} Q_{\epsilon\epsilon}^{-1} Q_{\epsilon x})x \). Thus,

\[
Q(x, \epsilon) - Q(x, \dot{\epsilon}) = \epsilon' Q_{\epsilon\epsilon} \epsilon + \epsilon' Q_{\epsilon x} x + x' Q_{x\epsilon} \epsilon + x' Q_{x\epsilon} Q_{\epsilon\epsilon}^{-1} Q_{\epsilon x} x
\]

\[
= \epsilon' Q_{\epsilon\epsilon} \epsilon - \epsilon' Q_{\epsilon\epsilon} \dot{\epsilon} - \dot{\epsilon}' Q_{\epsilon\epsilon} \dot{\epsilon}
\]

\[
= (\epsilon - \dot{\epsilon})' Q_{\epsilon\epsilon} (\epsilon - \dot{\epsilon}).
\]

As \( Q(x, \dot{\epsilon}) = \min_x Q(x, \epsilon) \) is a constant in the integral in equation (A.1), we find that

\[
\int \exp \left[ -\frac{1}{2} Q(x, \epsilon) \right] d\epsilon = \exp \left[ -\frac{1}{2} \min_x Q(x, \epsilon) \right] \times \int \exp \left[ -\frac{1}{2} (\epsilon - \dot{\epsilon})' Q_{\epsilon\epsilon} (\epsilon - \dot{\epsilon}) \right] d\epsilon.
\]

Therefore, the constant of proportionality in equation (A.1) is

\[
\int \exp(-\frac{1}{2} \Delta' Q_{\epsilon\epsilon} \Delta) d\Delta = (2\pi)^{m/2} \det(Q_{\epsilon\epsilon})^{-1/2},
\]

where \( m \) is the dimension of \( \epsilon \), and hence it is independent of \( x \). Then, suppose that we solve the program

\[
\min_x \int \exp \left[ -\frac{1}{2} Q(x, \epsilon) \right] d\epsilon.\]

Assume that \( Q \) admits a saddle point with respect to \( \epsilon \) and \( x \), so that \( \max_x \min_x Q(x, \epsilon) \) exists. From equation (A.1)

\[
\min_x \int \exp \left[ -\frac{1}{2} Q(x, \epsilon) \right] d\epsilon \propto \min_x \exp \left[ -\frac{1}{2} \min_x Q(x, \epsilon) \right] = \exp \left[ -\frac{1}{2} \max_x \min_x Q(x, \epsilon) \right].\]

It is worth noting this result applies also when \( Q \) is a non-homogeneous quadratic form, which depends on \( x \) and \( \epsilon \), alongside a third vector \( z \), insofar it admits a saddle point \( \max_x \min_x Q(x, \epsilon, z) \).

Then, let us introduce the Discounted Stress proposed by Vitale (2013).

**Definition 2** The (discounted) stress function in \( n \) is \( S_n \equiv c_n + e^{-r\Delta} \mathbf{V}_{n+1} - \frac{1}{p}(\epsilon_n)^2/\sigma_n^2 \).

**Proof of Lemma 1.**

Let assume that in \( n + 1 \) the optimization criterion \( \mathbf{V}_{n+1} \) is a quadratic form in \( g_n - \hat{g}_n \), so that two positive constants, \( B_{n+1} \) and \( C_{n+1} \), exist such that \( \mathbf{V}_{n+1} = -B_{n+1}(g_n - \hat{g}_n)^2 - C_{n+1} \). Because the exponential function is monotonic we have that

\[
\exp(\frac{\Delta}{2} \mathbf{V}_n) = \min_{x_n} E_n \left[ \exp (\frac{\Delta}{2} (c_n + e^{-r\Delta} \mathbf{V}_{n+1})) \right].
\]

Since i) \( g_n - \hat{g}_n \) is linearly dependent on \( \epsilon_n \) via equation (2.1), ii) \( c_n = (g_n - \hat{g}_n)x_n \) and iii) the optimization criterion \( \mathbf{V}_{n+1} \) is assumed to be a quadratic form in \( g_n - \hat{g}_n \), then the distribution of \( c_n + e^{-r\Delta} \mathbf{V}_{n+1} \) depends on that of \( \epsilon_n \) and hence, given that \( \epsilon_n \sim N(0, \sigma_n^2) \),

\[
\min_{x_n} E_n \left[ \exp \left( \frac{\Delta}{2} (c_n + e^{-r\Delta} \mathbf{V}_{n+1}) \right) \right] = (2\pi \sigma_n^2)^{-1/2} \min_{x_n} \int \exp \left( \frac{\Delta}{2} (S_n) \right) d\epsilon_n.
\]

Now, since \( \mathbf{V}_{n+1} \) is assumed to be a quadratic form in \( g_n - \hat{g}_n \) and this is linear in \( \epsilon_n, x_n \) and \( g_{n-1} - \hat{g}_{n-1} \), \( \mathbf{V}_{n+1} \) can be expressed as a quadratic form in \( \epsilon_n, x_n \) and \( g_{n-1} - \hat{g}_{n-1} \). Similarly, \( c_n \) is a quadratic form in \( \epsilon_n, x_n \) and \( g_{n-1} - \hat{g}_{n-1} \) and so is \( S_n \). Thus, if the stress in \( n \) admits a saddle point, in that \( \min_{x_n} \max_{\epsilon_n} S_n \) exists, then \( -S_n \) admits a saddle point in the statement of Lemma 4. Exploiting this
Lemma

\[
\min_{x_n} \int \exp \left( \frac{\rho S_n}{2} \right) d\epsilon_n = \min_{x_n} \int \exp \left( -\frac{1}{2} \frac{-\rho S_n}{Q(x_n, \epsilon_n)} \right) d\epsilon_n
\]

\[
= K_n \exp \left( \frac{1}{2} \max_{x_n} \min_{\epsilon_n} (-\rho S_n) \right) = K_n \exp \left( \frac{\rho}{2} \min_{x_n} \max_{\epsilon_n} S_n \right),
\]

where, using the result outlined in the proof of Lemma 4, we establish that \( K_n = (2\pi/q_{\epsilon_n})^{1/2} \) with \( q_{\epsilon_n} \) equal to the second derivative of \(-\rho S_n\) with respect to \( \epsilon_n \). This implies that

\[
\min_{x_n} E_n \left[ \exp \left( \frac{\rho}{2} (cn + e^{-r\Delta} V_{n+1}) \right) \right] = (\sigma^2 q_{\epsilon_n})^{-1/2} \exp \left( \frac{\rho}{2} \min_{x_n} \max_{\epsilon_n} S_n \right).
\]

This implies that extremizing \( S_n \), i.e. maximizing it with respect to \( \epsilon_n \) and minimizing the resulting function with respect to \( x_n \), we find that in period \( n \): i) the saddle point pins down the optimal market order for the insider; ii) the extremized stress, equal to the saddle point value \( \min_{x_n} \max_{\epsilon_n} S_n \), is a quadratic form in \( g_{n-1} - \hat{g}_{n-1}, -B_n(g_{n-1} - \hat{g}_{n-1})^2 - e^{-r\Delta} C_{n+1} \), and iii) because \( \exp(\frac{\rho}{2} V_n) = \min_{x_n} E_n \left[ \exp \left( \frac{\rho}{2} (cn + e^{-r\Delta} V_{n+1}) \right) \right] \), the optimization criterion \( V_n \) is a quadratic form in \( g_{n-1} - \hat{g}_{n-1} \) equal to the extremized stress plus a constant independent of \( g_{n-1} - \hat{g}_{n-1} \),

\[
V_n = -\gamma_n + \min_{x_n} \max_{\epsilon_n} S_n, \quad \text{where} \quad \gamma_n = \frac{1}{\rho} \ln(\sigma^2 q_{\epsilon_n} \epsilon_n).
\]

In general the saddle point for \( S_{n+j} \) must be derived in all future dates \( n+1, n+2, \cdots, n+j, \cdots \) before it can be found in \( n \) to determine the optimal market order \( x_n \). However, with a stationary trading strategy we simply need to find a fix point in the double recursion implied by the extermination of the stress. In fact, with a stationary strategy we see that \( B_n = B_{n+1} = B, C_n = C_{n+1} = C \) and \( \gamma_n = \gamma = \frac{1}{\rho} \ln(\sigma^2 q_{\epsilon_n} \epsilon_n) \), with \( q_{\epsilon_n} \epsilon_n = \frac{\rho}{2} + e^{-r\Delta} \rho B \). Then, we have \( V_n = -\gamma + \min_{x_n} \max_{\epsilon_n} \left\{ c_n - e^{-r\Delta} B(g_n - \hat{g}_n)^2 - e^{-r\Delta} C - \frac{1}{\rho} (\epsilon_n)^2/\sigma^2 \right\} \), so that a stationary trading strategy implies that the following fixed points hold,

\[
-B(g_{n-1} - \hat{g}_{n-1})^2 = \min_{x_n} \max_{\epsilon_n} \left\{ c_n - e^{-r\Delta} B(g_n - \hat{g}_n)^2 - \frac{1}{\rho} \frac{\Delta_n^2}{\sigma^2} \right\} \quad \text{and} \quad C = \gamma + e^{-r\Delta} C. \quad \Box
\]

**Proof of Lemma 2.**

Since we concentrate on stationary equilibria, we assume that \( V_{n+1} = -B(g_n - \hat{g}_n)^2 - C \), with \( B \) and \( C \) non-negative constants, and that the market maker sets the transaction price according to equation (1.4), where his conditional expectation of the asset’s underlying profitability is given by equation (1.6). Then, it follows that \( g_n - \hat{g}_n \) respects equation (2.1). We then apply Lemma 1. Taking the derivative of
\( \mathcal{S}_n \) with respect to \( \epsilon_n \) we have that as \( 1 + e^{-r \Delta} \rho \sigma^2_\xi B > 0 \),

\[
\epsilon_n^{\max} = \theta_y (g_{n-1} - \hat{g}_{n-1}) + \theta_x x_n,
\]

where

\[
\theta_y = -e^{-r \Delta} (1 - (\kappa + \nu \lambda_d) \Delta) \frac{\rho \sigma^2_\xi}{1 + e^{-r \Delta} \rho \sigma^2_\xi} B,
\]

\[
\theta_x = \frac{1}{2} (1 - 2e^{-r \Delta} \lambda_x B) \frac{\rho \sigma^2_\xi}{1 + e^{-r \Delta} \rho \sigma^2_\xi} B.
\]

Inserting into \( \mathcal{S}_n \) the expression for \( \epsilon_n^{\max} \) and minimizing with respect to \( x_n \) we find, after some tedious algebra, that if \( 4 \lambda_x (1 - e^{-r \Delta} \lambda B) + \rho \sigma^2_\xi > 0 \), a minimum is reached for \( x_n = \beta (g_n - \hat{g}_n) \), with

\[
\beta = \frac{(1 - (\kappa + \nu \lambda_d) \Delta) (1 - 2e^{-r \Delta} \lambda_x B)}{2 \lambda_x (1 - e^{-r \Delta} \lambda_x B) + \frac{1}{2} \rho \sigma^2_\xi}.
\]

Plugging this expression into \( \mathcal{S}_n \) after some long but straightforward algebra we find that \( \mathcal{V}_n = -B (g_{n-1} - \hat{g}_{n-1})^2 - C \), where

\[
B = \frac{(1 - 2e^{-r \Delta} \lambda_x B) (1 - (\kappa + \nu \lambda_d) \Delta)^2 \lambda_x}{4[(1 - e^{-r \Delta} \lambda_x B) \lambda_x + \frac{1}{4} \rho \sigma^2_\xi]^2} + e^{-r \Delta} \frac{(1 - (\kappa + \nu \lambda_d) \Delta)^2 \lambda_x^2}{4[(1 - e^{-r \Delta} \lambda_x B) \lambda_x + \frac{1}{4} \rho \sigma^2_\xi]^2} B
\]

\[
+ \frac{(1 - (\kappa + \nu \lambda_d) \Delta)^2}{16[(1 - e^{-r \Delta} \lambda_x B) \lambda_x + \frac{1}{4} \rho \sigma^2_\xi]^2} \rho \sigma^2_\xi,
\]

\[
C = -\gamma + e^{-r \Delta} C, \quad \text{where} \quad \gamma = -\frac{1}{\rho} \ln(\sigma^2_{\epsilon_n} q_{\epsilon_n} \epsilon_n) \quad \text{and} \quad q_{\epsilon_n} \epsilon_n = \frac{1}{\sigma^2_\xi} + e^{-r \Delta} \rho B.
\]

Simple manipulation of the latter equation gives the formulation for \( C \) in the statement, while rearranging the former implies that \( B \) solves the cubic equation

\[
\left(1 + \frac{1}{4 \lambda_x} \rho \sigma^2_\xi \right) - e^{-r \Delta} \lambda_x B \right) \left(4e^{-r \Delta} \lambda_x^2 B^2 - 4 \lambda_x B \left(1 + \frac{1}{4 \lambda_x} \rho \sigma^2_\xi \right) + (1 - (\kappa + \nu \lambda_d) \Delta)^2 \right) = 0.
\]

This possesses three roots. However, \( \left(1 + \frac{1}{4 \lambda_x} \rho \sigma^2_\xi \right) - e^{-r \Delta} \lambda_x B = 0 \) entails that \( 4 \lambda_x (1 - e^{-r \Delta} \lambda_x B) + \rho \sigma^2_\xi = 0 \) and hence it violates the second order condition of the minimization of the stress with respect to \( x_n \). The second part of this equation presents two roots,

\[
B_{\pm} = \frac{1}{2 \lambda_x} e^{-r \Delta} \left(1 + \frac{1}{4 \lambda_x} \rho \sigma^2_\xi \right) \pm \left[ \left(1 + \frac{1}{4 \lambda_x} \rho \sigma^2_\xi \right)^2 - e^{-r \Delta} (1 - (\kappa + \nu \lambda_d) \Delta)^2 \right]^{1/2} \right).
\]

Now consider that

\[
\beta \lambda_x = \frac{(1 - (\kappa + \nu \lambda_d) \Delta)(1 - 2e^{-r \Delta} \lambda_x B)}{2(1 - e^{-r \Delta} \lambda_x B) + \frac{1}{2} \lambda_x \rho \sigma^2_\xi}.
\]
Since, given the expression for \( \lambda_d \) in equation (1.7)
\[
1 - (\kappa + \nu \lambda_d) \Delta = (1 - \kappa \Delta) \frac{\Sigma_d \beta^2 \sigma_d^2 + \sigma_d^2 \sigma_f^2 \Delta}{\Sigma_d^2 (\beta^2 \sigma_d^2 + \nu^2 \sigma_f^2 \Delta^2) + \sigma_d^2 \sigma_f^2 \Delta} > 0,
\]
the condition \( \beta \lambda_x > 0 \) implied by equation (1.8) is equivalent to
\[
\frac{(1 - 2e^{-r \Delta} \lambda_x B)}{2(1 - e^{-r \Delta} \lambda_x B) + \frac{1}{2} \frac{1}{\lambda_x} \rho \sigma_e^2} > 0.
\]
It is immediate to check that for \( B_+ \) the numerator in this ratio is negative, while the denominator is positive, so that this constraint is violated. Instead, for \( B_- \) both numerator and denominator are positive and the constraint is satisfied. \( \Box \)

**Proof of Corollary 1.**

For \( \rho \downarrow 0 \)
\[
\beta \rightarrow \frac{(1 - (\kappa + \nu \lambda_d) \Delta)(1 - 2e^{-r \Delta} \lambda_x B)}{2\lambda_x (1 - e^{-r \Delta} \lambda_x B)},
\]
\[
B \rightarrow \frac{e^{r \Delta}}{2\lambda_x} \left( 1 - \left[ 1 - e^{-r \Delta} (1 - (\kappa + \nu \lambda_d) \Delta)^2 \right]^{1/2} \right),
\]
\[
C \rightarrow \frac{e^{-r \Delta}}{1 - e^{-r \Delta}} \sigma_e^2 B.
\]

As \( \sigma_e^2 = (\lambda_x^2 \sigma_d^2 + \sigma_d^2 + \lambda_x^2 \sigma_f^2) \Delta \) these expressions correspond to those derived by Chau and Vayanos. \( \Box \)

**Existence and Uniqueness of Solution to Equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8).**

We follow the route suggested by Chau and Vayanos. Plugging equation (2.6) into (2.5) and readjusting we find that
\[
\frac{\beta \lambda_x}{(1 - (\kappa + \nu \lambda_d) \Delta)} = \frac{\left[ (1 + \frac{1}{4 \lambda_x} \rho \sigma_e^2)^2 - e^{-r \Delta} (1 - (\kappa + \nu \lambda_d) \Delta)^2 \right]^{1/2}}{1 + \left[ (1 + \frac{1}{4 \lambda_x} \rho \sigma_e^2)^2 - e^{-r \Delta} (1 - (\kappa + \nu \lambda_d) \Delta)^2 \right]^{1/2}} + \frac{1}{4 \lambda_x} \rho \sigma_e^2.
\]

Plugging equations (1.7) and (1.8) into the left hand side of this equation we find that this is equal to
\[
\frac{\beta^2 \Sigma_g}{\beta^2 \Sigma_g + \sigma_f^2 \Delta},
\]
so an equilibrium value for \( \beta \) is found when it solves the following equation
\[
\frac{\beta^2 \Sigma_g}{\beta^2 \Sigma_g + \sigma_f^2 \Delta} = \frac{\left[ (1 + \frac{1}{4 \lambda_x} \rho \sigma_e^2)^2 - e^{-r \Delta} (1 - (\kappa + \nu \lambda_d) \Delta)^2 \right]^{1/2}}{1 + \left[ (1 + \frac{1}{4 \lambda_x} \rho \sigma_e^2)^2 - e^{-r \Delta} (1 - (\kappa + \nu \lambda_d) \Delta)^2 \right]^{1/2}} + \frac{1}{4 \lambda_x} \rho \sigma_e^2. \tag{A.2}
\]

Chau and Vayanos show that the ratio in the left hand side of equation (A.2) takes a value in the interval \([0,1]\), that it is continuous and monotonically increasing in \( \beta \) and it converges to 1 for \( \beta \uparrow \infty \) and 0 for
For \( \rho = 0 \) Chau and Vayanos show, substituting out equations (1.7) and (1.8) into \( 1 - (\kappa + \nu \lambda_d)\Delta \), that the right hand side is continuous in \( \beta \) and that for \( \beta \uparrow \infty \) and \( \beta \downarrow 0 \) it converges to values strictly inside the interval \([0,1)\). This implies that at least one solution to the system of equations exists. To show that it is unique, they show that for \( \rho = 0 \) the right hand side of equation (A.2) is monotonically decreasing in \( \beta \). To see this one notice that for \( \rho = 0 \) the right hand side is decreasing in \( 1 - (\kappa + \nu \lambda_d)\Delta \) but this is increasing in \( \beta \). To see the latter they employ equations (1.7) and (1.8) to write

\[
1 - (\kappa + \nu \lambda_d)\Delta = (1 - \kappa \Delta) \frac{\Sigma_y \beta^2 \sigma_t^2 + \sigma_t^2 \sigma_y^2 \Delta}{\Sigma_y (\beta^2 \sigma_t^2 + \nu^2 \sigma_t^2 \Delta^2) + \sigma_t^2 \sigma_y^2 \Delta}.
\]

(A.3)

Using equation (2.8) Chau and Vayanos show that the ratio in equation (A.3) is increasing in \( \beta \) and therefore the right hand side of equation (A.2) is decreasing in \( \beta \). For \( \rho > 0 \), while it remains true that the right hand side of equation (A.2) is decreasing in \( 1 - (\kappa + \nu \lambda_d)\Delta \) and that this is in decreasing in \( \beta \), we have to consider the impact of \( \beta \) on \( 1 + \frac{1}{\lambda} \rho \sigma_t^2 \) and of this on the right hand side of equation (A.2). Firstly, we notice that the right hand side of equation (A.2) takes values in the interval \([0,1)\). Secondly, we notice from equation (1.8) that, for \( \beta \downarrow 0 \) \( \Sigma_y \uparrow \Sigma_y \), \( \sigma_t^2 \rightarrow \sigma_t^2 \) and \( \lambda_x \rightarrow 0 \) (with \( \Sigma_y > \sigma_y^2 \Delta \) and \( \sigma_t^2 > \sigma_y^2 \Delta \)), while for \( \beta \uparrow \infty \) \( \Sigma_y \downarrow \sigma_y^2 \Delta \), \( \sigma_t^2 \rightarrow \sigma_y^2 \Delta \) and \( \lambda_x \rightarrow 0 \). This implies that for \( \beta \downarrow 0 \) and \( \beta \uparrow \infty \) \( 1 + \frac{1}{\lambda} \rho \sigma_t^2 \uparrow \infty \). Then, it is immediate to conclude that for \( \beta \downarrow 0 \) and \( \beta \uparrow \infty \) the right hand side of equation (A.3) converges to 0 and hence it is a non-monotonic function of \( \beta \). In other words, because the right hand side of equation (A.3) does not preserve the regularity conditions valid for \( \rho = 0 \), we cannot establish either that a solution to the system of equations (2.6), (2.7), (2.5), (1.7), (1.8) and (2.8) exists or that it is unique.

- **Proof of Proposition 2.**

Let us introduce \( b \) and \( S_y \) such that \( \beta = b \sqrt{\Delta} \) and \( \Sigma_y = S_y \sqrt{\Delta} \). Chau and Vayanos show that for \( \Delta \downarrow 0 \) equation (2.8) converges to \( S_y^2 b^2 = \sigma_y^2 \sigma_t^2 \). As for equation (A.2) it can also be written as follows

\[
\frac{S_y b^2}{S_y b^2 \sqrt{\Delta} + \sigma_t^2} = \left[ \frac{1}{\sqrt{\lambda}} \left( 1 + \frac{1}{\sqrt{\gamma}} \rho \sigma_t^2 \right)^2 - \frac{1}{\sqrt{\lambda}} e^{-r\Delta} (1 - (\kappa + \nu \lambda_d)\Delta)^2 \right]^{1/2} - \frac{1}{\sqrt{\lambda}} \frac{1}{\sqrt{\gamma}} \rho \sigma_t^2 \\
1 + \left[ \left( 1 + \frac{1}{\sqrt{\gamma}} \rho \sigma_t^2 \right)^2 - e^{-r\Delta} (1 - (\kappa + \nu \lambda_d)\Delta)^2 \right]^{1/2} + \frac{1}{\sqrt{\gamma}} \rho \sigma_t^2.
\]

(A.4)

Inspection of equation (A.3) allows to write \( 1 - (\kappa + \nu \lambda_d)\Delta = (1 - \kappa \Delta)(1 + o(\Delta^{1/2})) \) where \( o(\Delta^{1/2}) \) denotes a function of \( \Delta \) of order 3/2. Similarly, considering the definition of \( \sigma_t^2 \) and equation (1.8), we can write

\[
\frac{1}{\lambda} \sigma_t^2 = (\lambda_x \sigma_t^2 + \frac{1}{\lambda} \sigma_y^2) \Delta + o(\Delta^{3/2}).
\]

Exploiting these expressions we can write the right hand side of equation (A.4) as follows

\[
\left[ \frac{1}{\sqrt{\lambda}} \left( 1 - e^{-r\Delta} + 2 \kappa \Delta e^{-r\Delta} + \frac{1}{\sqrt{\gamma}} \rho (\lambda_x \sigma_t^2 + \frac{1}{\lambda} \sigma_y^2) \Delta + o(\Delta^{3/2}) \right)^2 \right]^{1/2} - \frac{1}{\sqrt{\lambda}} \rho (\lambda_x \sigma_t^2 + \frac{1}{\lambda} \sigma_y^2 + o(\Delta^{1/2})) \sqrt{\Delta} \\
1 + \left[ \left( 1 - e^{-r\Delta} + 2 \kappa \Delta e^{-r\Delta} + \frac{1}{\sqrt{\gamma}} \rho (\lambda_x \sigma_t^2 + \frac{1}{\lambda} \sigma_y^2) \Delta + o(\Delta^{3/2}) \right)^2 \right]^{1/2} + \frac{1}{\sqrt{\gamma}} \rho (\lambda_x \sigma_t^2 + \frac{1}{\lambda} \sigma_y^2 + o(\Delta^{1/2})) \Delta.
\]
Exploiting the Hôpital's rule and considering that for \( \Delta \downarrow 0 \), \( \lambda_x \) converges to \( \frac{S_b \Delta^2}{\sigma_d^2} \), we find that, the right hand side of equation (A.4) converges to \( 2\kappa + r + \frac{1}{2} \rho \frac{1}{S_g} (S_g^2 \sigma^2_t + \sigma^2_g) \), while the left hand side converges to \( \frac{S_g \Delta^2}{\sigma_d^2} \). In brief, we find that for \( \Delta \downarrow 0 \) equations (2.8) and (A.2) converge to the following

\[
S_g^2 \sigma^2_t = \sigma^2_g, \quad \frac{S_g b^2}{\sigma_d^2} = 2\kappa + r + \frac{1}{2} \rho \frac{1}{S_g} (S_g^2 b^2 + \sigma^2_g). 
\]

From these equations we see that

\[
\lim_{\Delta \downarrow 0} b = \left( 2\kappa + r + \rho \sigma_t \sigma_g \right)^{1/2} \frac{\sigma_t}{\sigma_g}, \quad \lim_{\Delta \downarrow 0} S_g = \frac{1}{\left( 2\kappa + r + \rho \sigma_t \sigma_g \right)^{1/2} \sigma_g^2}.
\]

In addition, \( \lim_{\Delta \downarrow 0} \lambda_x = \left. \lim_{\Delta \downarrow 0} \frac{S_g b}{\sigma_d^2} \right| = \frac{\sigma_d}{\sigma_g} \). As for \( \lambda_d \), notice that it can be written as

\[
\lambda_d = \frac{(1 - \kappa \Delta) \frac{S_g \nu \sigma_t \sigma_g^2 \sqrt{\Delta}}{S_g \sqrt{\Delta} (b^2 \sigma_d^4 + \nu^2 \sigma_t^2 \Delta) + \sigma_g^2 \sigma_t^2}}.
\]

so that \( \lim_{\Delta \downarrow 0} \frac{\lambda_d}{\sigma_x} = \lim_{\Delta \downarrow 0} \frac{S_g \nu \sigma_t \sigma_g^2 \sqrt{\Delta}}{S_g \sqrt{\Delta} (b^2 \sigma_d^4 + \nu^2 \sigma_t^2 \Delta) + \sigma_g^2 \sigma_t^2} \). In addition, since \( \lim_{\Delta \downarrow 0} \frac{1}{\lambda_x} \sigma_x = 0 \), \( \lim_{\Delta \downarrow 0} 1 - (\kappa + \nu \lambda_d) \Delta = 1 \) and \( \lim_{\Delta \downarrow 0} e^{-r \Delta} = 1 \), it is found that \( \lim_{\Delta \downarrow 0} B = \lim_{\Delta \downarrow 0} \frac{1}{2} \lambda_x = \frac{1}{2} \frac{\sigma_d}{\sigma_g} \). Finally, to determine the asymptotic behavior of \( C \) we apply the Hôpital's rule. In doing so, it should be considered that \( e^{-r \Delta} \sigma_x^2 = o(\Delta^{1/2}) \), while \( B e^{-r \Delta} \sigma_x^2 = B e^{-r \Delta} (\sigma_g^2 + \lambda_x^2 \sigma_t^2 + o(\Delta^{1/2})) \). Then, applying the Hôpital's rule, and considering that \( \lim_{\Delta \downarrow 0} \sigma_d^2 = 0 \), we find that

\[
\lim_{\Delta \downarrow 0} C = \frac{1}{\rho} \lim_{\Delta \downarrow 0} \frac{\rho B e^{-r \Delta} (\sigma_g^2 + \lambda_x^2 \sigma_t^2)}{re^{-r \Delta} (1 + e^{-r \Delta} \rho B \sigma_t^2)} = \frac{1}{r} \lim_{\Delta \downarrow 0} B (\sigma_g^2 + \lambda_x^2 \sigma_t^2) = \frac{1}{r} \frac{\sigma_g \sigma_t}{\sigma_g}. 
\]
Figure 1: Convergence to asymptotic values for the conditional variance of underlying profitability ($\Sigma_g$; top, left panel), the insider’s trading intensity ($\beta$; top, right panel), the market liquidity coefficient ($\lambda_x$; bottom, left panel) and the impact of dividend stream ($\lambda_d$; bottom, right panel) for $\sigma_l = 1, \sigma_g = 0.62, \sigma_d = 1.06, \nu = 1.47, \kappa = 0, r = 0.02$ and $\rho = 1$. 
Figure 2: Dependence of conditional variance of underlying profitability ($\Sigma_g$; top, left panel), the insider’s trading intensity ($\beta$; bottom, left panel), the market liquidity ($\lambda_x$; top, right panel) and the price impact of dividend ($\lambda_d$; bottom, right panel) on risk-aversion, $\rho$, for $\sigma_l = 1$, $\sigma_g = 0.62$, $\sigma_d = 1.06$, $\nu = 1.47$, $\kappa = 0$, $r = 0.02$ and four different trading frequencies.
Figure 3: Dependence of conditional variance of underlying profitability ($\Sigma_g$; top, left panel), half-life of prediction error of market maker ($t_{0.5}$; top, middle panel), half-life of prediction error of market maker relative to the benchmark scenario with no insider ($t_{0.5}/t_{0.5,0}$; top, right panel), the insider’s trading intensity ($\beta$; bottom, left panel), the market liquidity ($\lambda_x$; bottom, middle panel) and the price impact of dividend ($\lambda_d$; bottom, right panel) on risk-aversion, $\rho$, for $\sigma_g = 0.62$, $\sigma_d = 1.06$, $\nu = 1.47$, $\kappa = 0$, $r = 0.02$, two different trading frequencies ($\Delta = 1/252$ and $\Delta = 1/2016$) and volumes of liquidity trading ($\sigma_l^2 = 1$ and $\sigma_l^2 = 2$).
Figure 4: Dependence of conditional (left panel) and unconditional (right panel) optimization criterion on risk-aversion, $\rho$, for $\sigma_l = 1$, $\sigma_g = 0.62$, $\sigma_d = 1.06$, $\nu = 1.47$, $\kappa = 0$, $\gamma = 0.02$ and different trading frequencies.